Effect of Electron-Nuclear Spin Interaction on Electron-Spin Qubit Operations

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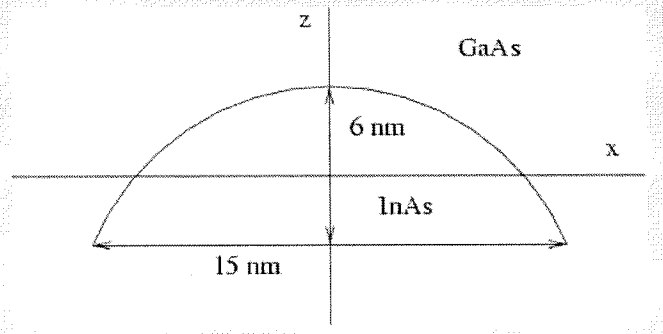
Question: Is a quantum-computer architecture, based on electron spins in InAs/GaAs quantum dots, scalable to many qubits?

Consideration: Inhomogeneous quantum-dot environments lead to the fluctuation of magnetic field (ΔB_N) from qubit to qubit.

Main results:

- ✓ Electron-nuclear spin interaction causes $\Delta B_N \sim 100$ G.
- ✓ Single qubit operation error $< 10^{-4}$, if $\Delta B_N^z < 0.1$ G and $\Delta B_N^{xy} < 10$ G.
- ✓ Double qubit operation error $< 10^{-4}$, if exchange energy $> 10^{-4}$ eV.

Electron-Nuclear Spin Interaction

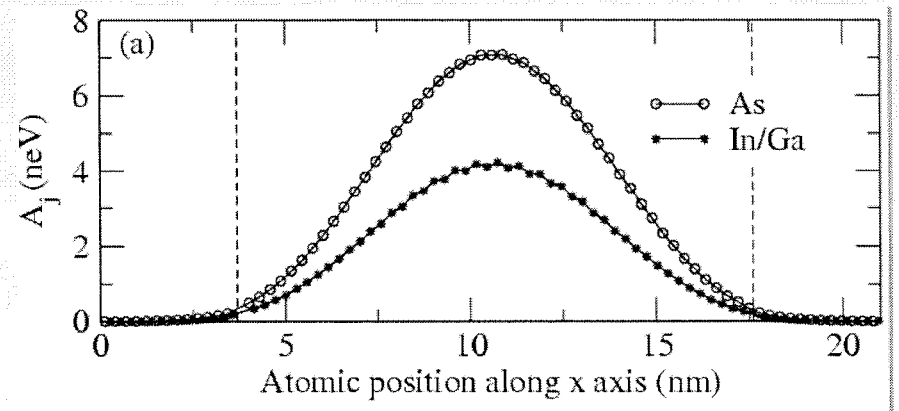


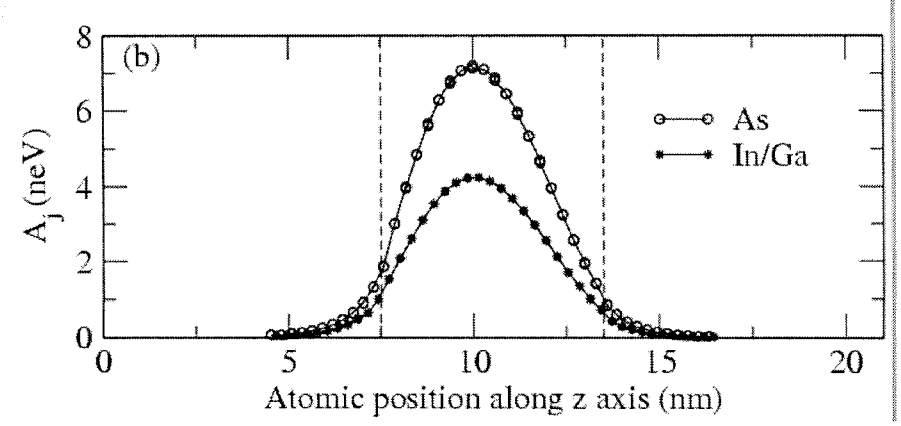
$$A_j = \frac{16\pi}{3}\mu_B \mu_j \left|\Psi(R_j)\right|^2$$

$$H_{HF} = \sum_j A_j I_j \cdot S \equiv g_e \mu_B B_N \cdot S$$

Effective Nuclear Magnetic Field

$$B_N = \frac{1}{g_e \mu_B}\sum_j A_j I_j$$



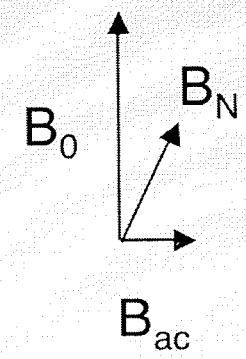


Spatial Fluctuation of Nuclear Magnetic Field

$$B_N = \frac{1}{g_e \mu_B} \sum_j A_j I_j, \qquad \Delta \vec{B}_N = \sqrt{\left\langle B_N^2 \right\rangle - \left\langle \vec{B}_N \right\rangle^2}$$

Environment	ΔB_N
Random nuclear spin configuration	100 G
Dot-size distribution (10%)	100 G
Alloy disorder ($In_{0.5}Ga_{0.5}As$)	10 G
Interface disorder (diffusion length = 1.2 nm)	0.1 G

Single Qubit Operation with ESR



Spin Precession Frequency

$$\omega_e = g_e \mu_B \sqrt{(B_0 + B_N^{\parallel})^2 + (B_N^{\perp})^2}$$

ESR Tuning Condition

$$\omega_e = \omega_{ac}$$

If B_N changes in time after gate calibration, ESR field is detuned. $\omega_e \neq \omega_{ac}$

Spin Hamiltonian

$$\begin{bmatrix} \hbar\omega_e/2 & \hbar\omega_B \cos\omega_{ac} t \\ \hbar\omega_B \cos\omega_{ac} t & -\hbar\omega_e/2 \end{bmatrix},$$

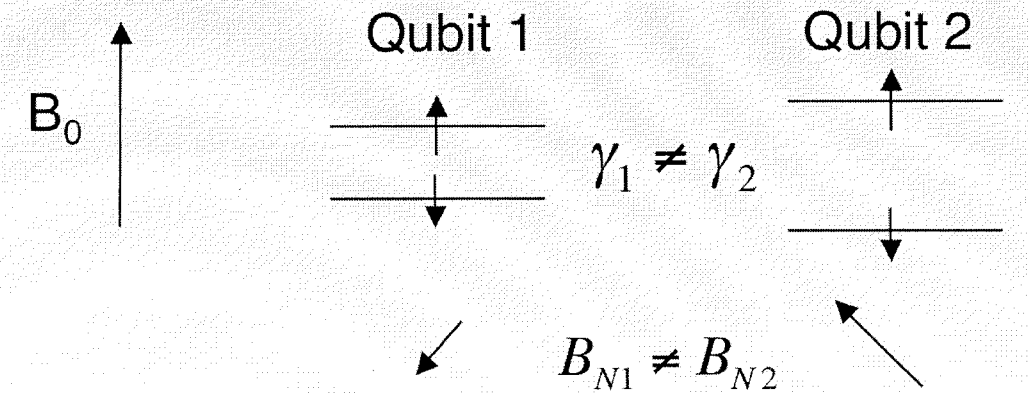
where $\omega_B = g_e \mu_B B_{ac}/2$

Spin π rotation

$$|\uparrow\rangle \xrightarrow{\omega_B t = \pi/2} \sqrt{\frac{(\omega_{ac} - \omega_e)^2}{(\omega_{ac} - \omega_e)^2 + \omega_B{}^2}}|\uparrow\rangle + \sqrt{\frac{\omega_B{}^2}{(\omega_{ac} - \omega_e)^2 + \omega_B{}^2}}|\downarrow\rangle$$

Rotation Error: $(\omega_e - \omega_{ac})^2 / \omega_B{}^2$

Double Qubit Operation with Exchange Int.



Spin Hamiltonian

$$\begin{bmatrix} \gamma_1+\gamma_2 & \gamma_1 & \gamma_2 & 0 \\ \gamma_1 & \gamma_1-\gamma_2 & J/2 & -\gamma_2 \\ \gamma_2 & J/2 & \gamma_2-\gamma_1 & -\gamma_1 \\ 0 & -\gamma_2 & -\gamma_1 & -\gamma_1-\gamma_2 \end{bmatrix}$$

Swap Operation

$$|\uparrow\downarrow\rangle \xrightarrow{4Jt=\pi} \sqrt{\frac{1}{1+x^2}}|\downarrow\uparrow\rangle + \sqrt{\frac{x^2}{1+x^2}}|\uparrow\downarrow\rangle$$

$$x = \frac{\gamma_1-\gamma_2}{2J}$$

Swap Error : $(\gamma_1-\gamma_2)^2/J^2$

Qubit Operation Error under Threshold

Since quantum error correction code can fix error $\varepsilon < 10^{-4}$,

1) Single Qubit Operation Error

$$\varepsilon = (\Delta B_N^{\parallel} + \Delta B_N^{\perp} \frac{B_N^{\perp}}{B_0})^2 / B_{ac}{}^2 < 10^{-4},$$

if $\Delta B_N^{\parallel} < 10^{-5}$ T and $\Delta B_N^{\perp} < 10^{-3}$ T

when $B_0 \sim 1$ T, $B_{ac} \sim 10^{-3}$ T, and $B_N^{\perp} \sim 10^{-2}$ T.

2) Double Qubit Operation Error

$$\varepsilon = (\gamma_1 - \gamma_2)^2 / J^2 < 10^{-4},$$

if $J > 10^{-4}$ eV when $\gamma_1 - \gamma_2 \sim 10^{-6}$ eV