Electron-Spin Qubit Operation Errors due to Electron-Nuclear Spin Interaction

quant-ph/0403122, funded by ARDA

Seungwon Lee, P. von Allmen, F. Oyafuso, G. Klimeck
NASA Jet Propulsion Laboratory, California Institute of Technology
K. B. Whaley
University of California, Berkeley

Question: Is a quantum-computer architecture, based on electron spins in InAs/GaAs quantum dots, scalable to many qubits?

Consideration: Inhomogeneous quantum-dot environments lead to the fluctuation of magnetic field (ΔB_N) from qubit to qubit.

Results:

- ✓ Electron-nuclear spin interaction causes $\Delta B_N \sim 100$ G.
- ✓ Double qubit operation error $< 10^{-4}$, if exchange energy $> 10^{-4}$ eV.

Electron-Nuclear Spin Interaction with Tight-Binding Wave Functions

$$H_{HF} = \sum_j A_j I_j \cdot S \equiv g_e \mu_B B_N \cdot S$$

Hyperfine Coupling Constant

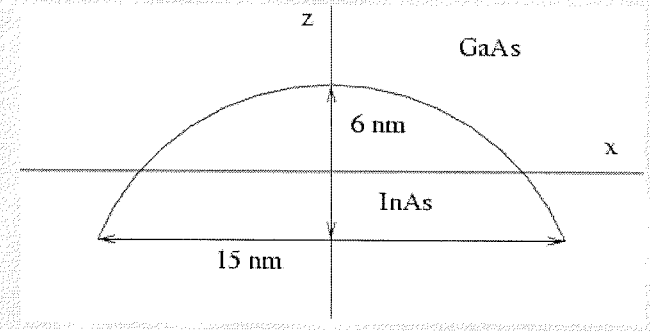
$$A_j = \frac{16\pi}{3} \mu_B \mu_j g_j \left|\Psi(R_j)\right|^2$$

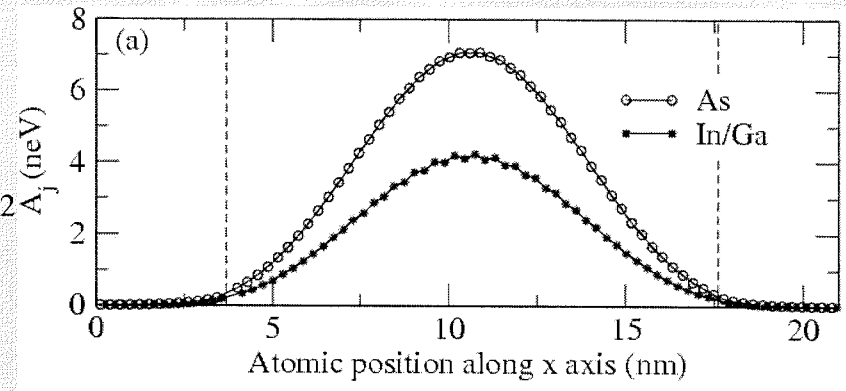
$$= \frac{16\pi}{3} \mu_B \mu_j g_j \left|\alpha_j \phi_s(R_j) + \beta_j \phi_{s*}(R_j)\right|^2$$

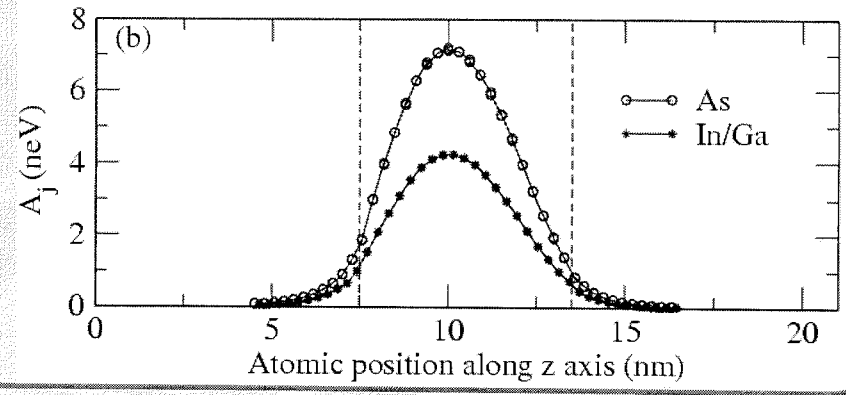
Tight-Binding Coefficient

Tight-Binding Orbital

Overhauser Experiment





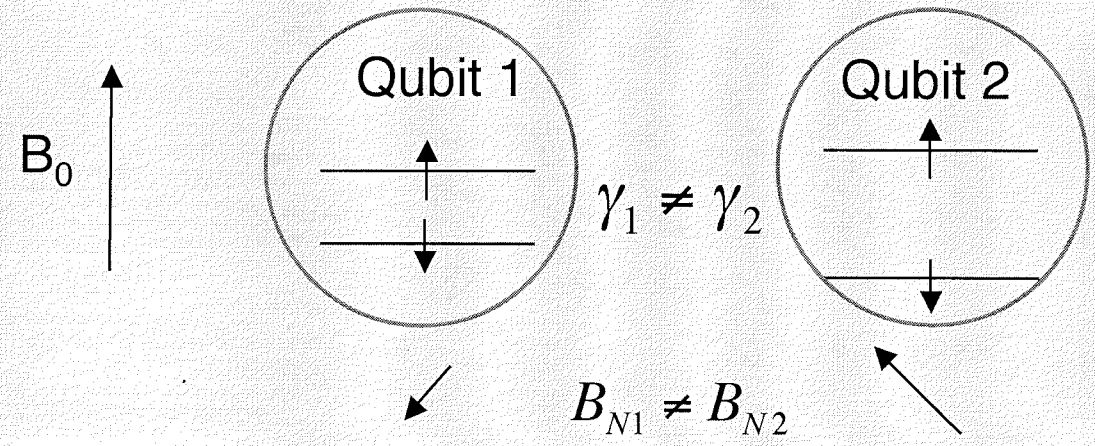


Spatial Fluctuation of Nuclear Magnetic Field

$$B_N = \frac{1}{g_e \mu_B} \sum_j A_j I_j, \qquad \Delta B_N = \sqrt{\langle B_N^2 \rangle - \langle B_N \rangle^2}$$

Environment	ΔB_N
Random nuclear spin configuration	100 G
Dot-size distribution (10%)	100 G
Alloy disorder ($In_{0.5}Ga_{0.5}As$)	10 G
Interface disorder (diffusion length = 1.2 nm)	0.1 G

Double Qubit Operation with Exchange Int.



Spin Hamiltonian

$$\begin{array}{cccc} \uparrow\uparrow & \uparrow\downarrow & \downarrow\uparrow & \downarrow\downarrow \end{array}$$
$$\begin{bmatrix} \gamma_1+\gamma_2 & \gamma_1 & \gamma_2 & 0 \\ \gamma_1 & \gamma_1-\gamma_2 & J/2 & -\gamma_2 \\ \gamma_2 & J/2 & \gamma_2-\gamma_1 & -\gamma_1 \\ 0 & -\gamma_2 & -\gamma_1 & -\gamma_1-\gamma_2 \end{bmatrix}$$

Swap Operation

$$|\uparrow\downarrow\rangle \xrightarrow{4Jt=\pi} \sqrt{\frac{1}{1+x^2}}|\downarrow\uparrow\rangle + \sqrt{\frac{x^2}{1+x^2}}|\uparrow\downarrow\rangle$$

$$x = \frac{\gamma_1-\gamma_2}{2J}$$

Swap Error : $(\gamma_1-\gamma_2)^2/J^2$

Hu, PRL 86, 918

Double Qubit Operation Error

- Quantum error correction code can fix error $\varepsilon < 10^{-4}$
- Double qubit operation error due to B_N spatial fluctuation:

$$\varepsilon = (\gamma_1 - \gamma_2)^2 / J^2 < 10^{-4}$$

$\gamma_1 - \gamma_2 \sim 10^{-6}$ eV,
since $\Delta B_N \sim 100$ G

If $J > 10^{-4}$ eV

- Error due to ΔB_N can be fixed if J > 10^{-4} eV is used.
- J<< ΔE (energy spacing) ~ 10^{-1} eV: qubit leakage prevented.

Effect of Electron-Nuclear Spin Interaction on Electron-Spin Qubit Operations

quant-ph/0403122, funded by ARDA

Seungwon Lee, P. von Allmen, F. Oyafuso, G. Klimeck
NASA Jet Propulsion Laboratory, California Institute of Technology
K. B. Whaley
University of California, Berkeley

Question: Is a quantum-computer architecture, based on electron spins in InAs/GaAs quantum dots, scalable to many qubits?

Consideration: Inhomogeneous quantum-dot environments lead to the fluctuation of magnetic field (ΔB_N) from qubit to qubit.

Results:

- ✓ Electron-nuclear spin interaction causes $\Delta B_N \sim 100$ G.
- ✓ Double qubit operation error $< 10^{-4}$, if exchange energy $> 10^{-4}$ eV.