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The orbit of Phoebe from Earthbased and Voyager observations

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Abstract. This article presents the results of a fit of a numerically integrated orbit for the Saturnian satellite Phoebe to Earthbased astrometric observations (from 1904 to 1996) and imaging data acquired by the Voyager 2 spacecraft during its encounter with Saturn. The primary results are the epoch state vector used in the integration and a set of mean elements which provide a geometrical representation of the orbit. We also assess the quality of the fit and the accuracy of the orbit.

Key words: Ephemerides Planets and satellites: general Planets and satellites:individual: Phoebe Solar system: general

1. Introduction

The ninth satellite of Saturn, Phoebe, was discovered by W. H. Pickering in March 1899 from photographs made in 1898 at the Harvard observatory station, Arequipa, Peru (Pickering 189(I). He subsequently published preliminary ephemerides for the satellite (Pickering 1905a,1905b). Ross (1905) produced the first definitive orbit based on observations made at Arequipa, Yerkes, and Lick in the early 1900's. Zadunaisky (1954) updated Ross' orbit using observations made between 1907 and 1942, Rose (1979) generated the first numerically integrated Phoebe orbit which was fit to observations over the period 1904 to 1969. He omitted those from Arequipa. Subsequent integrations were done by Bec-Borsenberger and Rocher (1982) who fit observations from 1904 to 1981 and by Bykova and Shikhalev (1982,1(184) who fit observations from 1898 to 1981.

The purpose of this work is to update the orbit in light of recent astrometric observations and imaging observations obtained by the Voyager 2 spacecraft. Futher incentive is provided by NASA's Cassini mission to Saturn (Kohlhase 1993). Current plans call for the Cassini spacecraft to fly by Phoebe at a distance of about 50000" km. A high quality ephemeris is needed to support scientific observations during the flyby.

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2. Orbit model

Our model for Phoebe's orbit is a numerical integration of its equation of motion (Peters 1981) which includes the effects of an oblate Saturn, perturbations from seven of the eight major Saturnian satellites (Hyperion, whose mass is unknown but presumed quilt small, is neglected), and perturbations from the Sun, Jupiter, and Uranus. The formulation is in Cartesian coordinates centered at the Saturnian system barycenter and referenced to the Earth mean equator and equinox of J2000 system. Phoebe is assumed to be massless, hence the barycenter location depends only upon the planet and perturbing satellites. JPL planetary ephemeris DE403 (Standish et al. 1995) provides the masses and positions of the Sun and perturbing planets, and JPL satellite ephemeris SAT077 (Jacobson 1996a) provides those of the perturbing satellites.

The model is far more complete than necessary to fit the observations. Rose's original work included only solar perturbations, that of Bykova and Shikhalev accounted for the Sun and Jupiter, and that of Bec-Bosenberger and Rocher took into account the effects of Titan, Jupiter, and the Sun. Phoebe's acceleration due to Jupiter is about 1% that of the Sun and due to Titan is about 0.1%. All three of these investigations obtained reasonable fits to the observations. We selected the model in this work primarily for consistency with the integrations of the other Saturnian satellites being performed in preparation for support of the Cassini mission (Jacobson 1996b). We did, however, also investigate a simplified model which replaces the perturbing satellites, except for Titan, with uniform circular equatorial rings retaining only their quadrupole effect (see Roy et al. 1988 for a discussion of this technique). In addition, the simplified model obtains its Titan positions from a precessing ellipse approximate representation of the Titan ephemeris (the elements for the ellipse can be found

in the Appendix). A comparison of integrations with the simplified and complete model found differences of" less than 50 km Over a 50 year' period. Ill fact, because such differences are weII below the accuracy of the pre-1966 observations (the integration epoch is November 11, 1966), we actually used the simplified model for all processing of those early observations.

The integration was carried out with a variable step sire, variable order, Gauss-Jackson method. An absolute truncation error limit of 10^{-10} km s^{-1} imposed 011 the velocity controlled the integration step. The average step size was 25 896 seconds and the maximum order was 15.

3. Observations

3. 1. Sources and summary

The literature search of Pierce (1975) lists publications containing micrometric and photographic Phoebe observations over the period 1898 to 1969. In their paper Bec-Borsenberger and Rocher extend the list to include photographic observations from 1975 to 1981. In addition, they provide several photographic observations made at La Sills in 1981 which apparently have not been published elsewhere. The only published observations found by the author subsequent to 1981 are those made at La Silla in 1982 (Debehogne 1984). The 11. S. Naval Observatory (Rohde 1 994) and McDonald Observatory (Whipple 1992, 1995) have supplied recent unpublished photographic observations, and McDonald Observatory (Whipple 1996) has supplied unpublished CCD observations.

During its encounter with Saturn in 1981, the Voyager 2 spacecraft acquired eight images of Phoebe against a star' background. These provide measures of Phoebe's position with accuracies in the 5010 150 km range.

Table 1 summarizes of all of the currently available Earthbased Phoebe observations grouped into sets. The first column contains the year of the observations; the second and third columns identify the observatory and instrument. A reference or publication is given in column four (see the References section of this paper). Finally, the code number in the last column uniquely indentifies the observation set.

3.2. Comments

All of the Arequipa observations are photographic. The initial reduction of sets 1-4 (Pickering 1905a) provided Saturn relative positions 'as measured on the image of the reticule' which all subsequent analyses appear to treat as equator of date positions. Pickering (1 905b) corrected 17 of the positions and reported them together with additional 1904 observations, the latter in the form of angular separation and declination difference between Phoebe and Saturn. The 1898 to 1902 photographs were later reduced against the Cape Photographic Durchmusterung, epoch 1875, star catalogue (Pickering 1908) yielding mean 1875.0 astrometric positions. For the 1905 and 1906 observations (Pickering 1906a, 1 906b, 1906c), Saturn-relative positions are again given in terms o{ angular separation and declination difference. According to the observers the precision of the entire set of Arequipa observations ranges from a few seconds of arc to nearly one minute.

The observations at Yerkes made by Barnard are \'isual micrometer observations of Phoebe relative to stars. The star positions were used to reduce the Phoebe observations to apparent place. The mean places of the comparison stars are also provided, and improved star positions could possibly improve the reduction. Notes indicate that the places for two of the stars are from the Astronomische Gesellschaft Catalogue and for the others are from the B.J. Fundamental Catalogue. Meridian circle measures were made for two of the 1906 comparison reference stars. For the 191 2/13 observations, meridian circle measures were made for one of the stars.

The observations at Lick made by Perrine and Albrecht are photographic. Those for set 7 were reduced using six catalogue stars to obtain 1904.0 mean places. The positions of the catalogue stars were taken from the Washington A.G. Zone catalogue; their 1904.() mean places are given in the reference. A Saturn observation is quoted but is too poor to use to reduce star catalogue errors. For the Albrecht set 9, 1905.() mean places are given, and for the Perrine sets 10 and 15 the respective mean places are for 1906.0 and 1908.(). It is presumed that all were reduced in the same manner as those of set 7, and it is probable that the Washington A .G . Zone catalogue was used in the reduction. Unfortunately, the positions of the comparison stars are not provided.

The Greenwich observations are photographic measures reduced to geocentric apparent positions using comparison stars measured relative to reference stars in the Astronomische Gesellschaft Catalogue. The publications include a correction for star catalog errors based 011 observations of Saturn reduced in the same manner as those of Phoebe and then compared to the tabular position of Saturn in the Nautical Almanac. Subtracting these corrections from the published observations prior to processing, in effect, replaces the star catalog error with the Saturn ephemeris error plus the error in measuring Saturn's position. A comparison of the Saturn tabular positions with ones from DE 403 found small differences relative to the size of the corrections.

The van Biesbroeck observations at Yerkes are photographic. The set 21 observations were reduced using comparison stars from the Abbadia catalogues and were reported as apparent positions. Set 25 was reduced with comparison stars from the Tacubaya catalogue and was reported in the form of astrometric positions for 1955.(),

The Mt. Wilson photographic observation of Nicholson and Richmond appears with a set of outer Jovian satellite observations made during the same time period. It is reported as an astrometric position for 1940.() with an

Table 1. Earthbased astrometric observation sets

Year	Observatory	Instrument	Reference	Code
1898	Arequipa	24in Bruce	Pickering(1908)	1
1899	Arequipa	24in Bruce	Pickering(1908)	2
1900	Arequipa	24in Bruce	Pickering(1908)	3
1902	Arequipa	24in Bruce	Pickering(1908)	4
1904	Arequipa	24in Bruce	Pickering(1905a,1905b)	5
1904	Yerkes	40in refactor	Barnard(1905)	6
1904	Lick	Crossley reflect or	Perrine(1904)	7
1905	Arequipa	24in Bruce	Pickering(1906a)	8
1905	Lick	Crossley reflector	Albrecht & Smith (1909)	9
1905	Lick	Crossley reflector	Perrine(1909)	10
1906	Arequipa	24in Bruce	Pickering(1906b)	11
1906	Arequipa	24in Bruce	Pickering(1906c)	12
1906	Yerkes	40in refactor	Barnard(1906)	13
1907	Greenwich	30in reflector	Christie(1907)	14
1908	Lick	Crossley reflector	Perrine(1909)	15
1908	Greenwich	30in reflector	Christie(1908)	16
1909	Greenwich	30in reflector	Christie(1909)	17
1910	Greenwich	30in reflector	Christie(1910)	18
1912/13	Yerkes	40in refactor	Barnard(1913)	19
1913	Yerkes	40in refactor	Barnard(1914)	20
1922	Yerkes	24in reflector	van Biesbroeck(1922)	21
1940	Mt. Wilson	100in reflector	Nicholson & Richmond(1944)	22
1942	McDonald	82in reflector	van Biesbroeck(1944)	23
1952	Cordoba	Normal astrograph	Bobone(1953)	24
1955	Yerkes	24in reflector	van Biesbroeck(1956)	25
1955	McDonald	82in reflector	van Biesbroeck(1956)	26
1957	Bloemfontien	ADH telescope	van Biesbroeck(1958)	27
1960	Flagstaff	40in reflector	Roemer & Lloyd(1966)	28
1968	Crimean	40cm astrograph	Chernykh & Chernykh(1971)	29
1969	Kitt Peak	21 3cm reflector	van Biesbroeck, et al. (1976)	30
1969	Catalina	154cm reflector	van Biesbroeck, et al. (1976)	31
1975/76	McDonald	2.1m reflector	Mulholland & Shelus(1980)	32
1981	Lowell	0.33m reflector	Bowell(1981/82)	33
1981	La Sills	40cm GPO	Debehogne(1981a,b,c,d/82)	34
1982	Lowell	0.33m reflector	Bowell(1988)	35
1982	La Silla	40cm GPO	Debehogne(1984)	36
1992	USNO	24in reflector	Rohde(1994)	37
1992	McDonald	2.1m reflector	Whipple(1992)	38
1993	USNO	24in reflector	Rohde(1994)	39
1994	McDonald	2.1m reflector	Whipple(1995)	40
1995/96	McDonald	2.1m reflector	Whipple(1996)	41

erroneous sign on the declination. Since the publication states that the Jovian satellite observations were reduced with the Astrographic Catalogue, it is presumed that the Phoebe observation was as well.

The observations by van Biesbroeck made at McDonald are photographic. Those in set 23 were reported as astrometric positions for 1942.0. They were probably reduced with comparison stars from the Tacubaya astrographic catalogue, although the reference simply states the 'astrographic catalogue'. Set, 26 was reduced against the Tacubaya astrographic catalogue and was reported in the form of astrometric positions for 1955,(). "

Bobone's Cordoba observations are photographic astrometric positions for 1952.0. Notes suggest that the reference stars for the reduction were from the Yale 17 and Yale 21 catalogues and that proper motion was not applied.

The photographic observations from Bloemfontien published by van Biesbroeck were actually made by T. Gehrels. They were reported as astrometric positions for 1957.(). There is no indication of the star catalogue used

in the reduction, but since it appears the plates were measured at Yerkes, it likely that the Tacubaya astrographic catalogue was used.

The Flagstaff observations of Roemer and Lloyd are photographic 1950.0 astrometric positions. They were made as part of a program to observe minor planets and faint natural satellites. Reference stars for the reduction were from the Astrographic Catalog.

The photographic observations in Set 29 made at the Crimean Astrophysical Observatory are reported as 1950.0 astrometric positions. No information is available about the star catalogue used in the reduction. Simultaneous observations of Hyperion and Iapetus are also reported. However, the differential positions of Phoebe relative to those bodies exhibit significant errors in the right ascension.

The Kitt Peak National Observatory photographic observation was made by van Biesbroeck and reduced by C.D. Vesely. It was reported as a 1950.0 astrometric position. An earlier publication suggests the Astrographic Catalog was the source of the reference stars in the reduction.

The photographic observations at the Catalina Station of LPL, made by van Biesbroeck and reduced by C.D. Vesely, were reported as 1950.0 astrometric positions. As in the case of the Kitt Peak observations, it appears that the Astrographic Catalog provided the reference stars. The September observations were noted to be outside the triangle of reference stars; they exhibit large residuals against the current Phoebe orbit.

The McDonald photographic observations of Mulholland and Shelus are 1950.0 astrometric positions. They were reduced using the USNO Zodical Catalogue. A number of simultaneous observations of other satellites are also given permitting the use of satellite relative positions rather than absolute positions.

Bowell's photographic observations at Lowell are 1950.0 astrometric positions. No indication of the star catalogue used in the reduction is given. Simultaneous observations of other satellites are also provided which permit the construction of satellite relative positions.

The photographic observations made by Debehogne at La Sills are reported as 1950.0 astrometric positions. As in the case of sets 32, 33, and 35, simultaneous observations of other satellites are given and can be used to form satellite relative observations. Notes in the publications and related publications (on Jovian and Uranian satellites) suggest the SAO catalogue was used in the reduction and that reference stars were corrected for proper motion.

Rohde's USNO photographic observations are J2000 geocentric astrographic positions. Set 37 has a significant bias in right ascension and set 39 has a significant bias in declination. Until the sources of these biases are understood, the observations are not usable. The Guide Star Catalogue provided the reference stars for the reduction.

The McDonald photographic observations of Whipple are J2000 topocentric astrographic positions. Reduction of sets 38 and 40 was made against the Astrographic Catalogue Reference Stars (ACRS) Catalogue. Since the positions of these stars are in FK4/B1950 system, they were rotated to FK5/J2000 system following the IAU Commission 20 conversion procedure (Standish et al. 1992). The McDonald CCD observations, set 41, are also J2000 astrographic and were reduced against the Guide Star Catalogue.

The Voyager observations are the pixel and line locations of the images of Phoebe and background reference stars in the Voyager camera frame. The 1'1{4/111950 star positions are also provided. These positions are from a special star catalogue (Klemola et al. 1979) made for the Voyager project and referenced to the Perth 70 catalogue. To facilitate processing of the observations the star positions were rotated to FK5/J2000 system following the IAU Commission 20 conversion procedure.

4. Orbit determination

4.1. Observation modeling

The determination of the orbit employs an algorithm which minimizes the sum of squares of the actual-minus-computed observation residuals. This procedure requires the formation of computed observables. In our approach, we attempt to compute the values of the observables as actually reported rather than transform the values to a uniform system (e.g. B1950 or J2000 system) as other authors have done. Because the integration is performed using the DE403 planetary ephemeris and SAT077 satellite ephemeris for perturbing body positions, the Phoebe orbit is in fact generated in the J2000 reference frame of the International Earth Rotation Service (IERS). The observation processing software also refers the Earth orientation (observer location) to that same frame.

The first step in computing the observables is the calculation of the *direction* from the observer to Phoebe in the IERS/J2000 system. The *natural direction* is used for astrographic and astrometric observations, and *proper direction* is used for apparent observations (see Murray 1983). For observations referred to the FK5/J2000 system, we form computed observables directly from the IERS/J2000 *direction*; the frame tie between the two systems is presumed to be much better than the error in the star catalogues used in the reduction of the observations. For observations referred to a mean equator of epoch system (this includes the FK4/B1950 system), we first precess the IERS/J2000 *natural direction* to the mean equator at the time of the observation with the IAU76 precession, then precess it from that time to the mean equator of epoch with the Newcomb precession. In addition, we make corrections for the FK4-FK5 equinox offset and elliptic aberration. For observations referred to a true-of-

date system (e.g. apparent positions), we apply the IAU76 precession and the 1980 IAU nutation to the IERS/J2000 *proper direction*. We also correct for the FK4-FK5 equinox offset. The Greenwich observations, however, are not corrected for equinox offset as the catalogue correction made to them is assumed to account for it. At the time of the Yerkes visual observations, the FK4-FK5 offset is quite small relative to the measurement error; hence, we decided for convenience to neglect the offset correction when processing them.

4.2. Observation processing

For each astrometric observation set, we used Householder transformations (Lawson and Hanson 1974) to pack the matrix of weighted observation partial derivatives and the weighted residual vector (actual minus computed observations) into an upper triangular square root information matrix and associated residual vector. This matrix and vector constitute the square root information array which is equivalent to the normal equations. Each column of the matrix and each element of the vector are associated with an epoch state vector component. Weights assigned to each observation set were based on our subjective assessment of the relative quality of the data in the set; they are proportional to the inverse of the error associated with the set. Within each set we often assigned two separate weights: one for the $\Delta\alpha$ or $\Delta\alpha\cos\delta$ or angular separation and the other for δ or $\Delta\delta$. We made numerous trial solutions with various weighting schemes and various data sets before arriving at the final weights. The Voyager images were processed with JPL's Optical Navigation and Orbit Determination Software to also obtain a square root information array. We set the weights for the image locations to correspond to a 1σ error of 0.5 pixel. It was assumed that no corrections were needed in the Voyager 2 trajectory; the trajectory was the same one used in the SAT077 major satellite ephemeris development. The square root information array for the complete data set was formed by combining the separate information arrays via Householder transformations. The solution for the state vector was generated and analyzed by means of singular value decomposition techniques (Lawson and Hanson 1974) applied to the composite square root information array.

5. Processing results

5.1. Observation residuals

Tables 2 and 3 give the postfit statistics for the observed minus computed residuals for the Voyager and Earthbased observations, respectively. The latter are grouped by data set, and for each set the table indicates the type of observations, the number of observations used versus the total number available, the weight for each type, and the mean and standard deviation of the residuals. The Arequipa, the 1904 Yerkes, and the USNO observations were not used; however, statistics for them are included. In arriving at the statistics for the unused observations, all residuals greater than 30″ were deleted and a 3σ rejection criterion was applied to the remainder.

Table 2. Voyager imaging residual statistics

Object	No.	Pixel Mean	Pixel Sigma	Line Mean	Line Sigma
Phoebe	8	0.042	0.241	0.146	0.281
Stars	13	-0.025	0.269	-0.090	0.224
All	21	0.001	0.255	0.000	0.267

overall, the orbit fits the Earthbased observations at the 1″.23 level. The RMS for the 'old' (pre-1940) sets is 1″.62 and for the 'modern' (post-1940) sets is 0″.88. Among the sets used, the poorest fit is to Barnard's 1906 micrometer measures, and the best fits are to Whipple's 1992 photographic data, Whipple's 1996 CCD data, and Mulholland's 1975 relative photographic data.

Figure 1 displays the right ascension and angular separation residuals for the observations used in the fit, and Figure 2 displays the declination residuals. The figures give an indication of the time distribution of the observations and show the overall quality of the fit. Considerable scatter is evident as well as some suggestion of systematic errors.

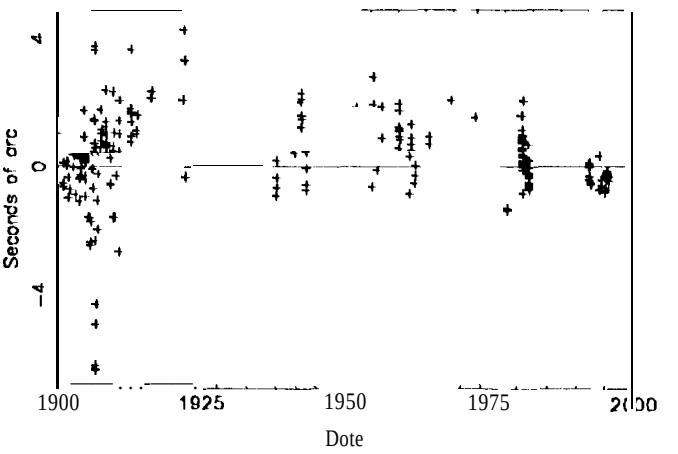

Fig. 1. Right ascension and angular separation residuals

5.2. Epoch state vector

The epoch state vector obtained from the fit appears in Table 4, and the Saturnian system dynamical constants needed for the integration appear in Table 5. Two state

Table 3. Astrometric observation residual statistics. The column entitled 'Type' gives the observation type: the letter V denotes visual, 1' denotes photographic, and C denotes CCD. The digit gives measurement type: o absolute α and δ; 1 $\Delta\alpha \cos\delta$ and $\Delta\delta$; 2 $\Delta\alpha$ and $\Delta\delta$; 4- angular separation and $\Delta\delta$. The column entitled 'No. 'gives the number of observations included in the solution/number of available observations, and the column entitled 'Wgt.' contains the weight assigned to the data set.

Set	Year	Olmrvcr/Source	Typc	No.	Mean	Sig.	Wgt .	No.	Mean	Sig.	Wgt
1	1898	Pickering	1'0	7/7	-0".44	4'."20	0	7/7	2".65	3".76	0
2	1899	Pickering	P0	0/3			0	0/3			0
3	1900	Pickering	P0	20/26	8".27	6".28	0	20/26	3'."17	5".87	0
4	1902	Pickering	1'0	2/6	.]]".2	19".8	0	2/6	-3".13	0!'31	0
5	1904	Pickering	1'4	26/28	6".4(i	4".53	0	26/28	-3','75	7".42	0
6	1904	Barnard	V0	1/2	21".7	0".00	0	1/2	-4".20	0".00	0
7	1904	Perrine	1'0	5I6	0'!31	0".99	100	5/6	-0".04	0".58	167
8	1905	Pickering	1'4	11/17	-2'."37	12':1	0	11/17	-4".47	14".2	0
9	1905	Albrecht	1'0	11/11	-1 '."04	0".97	67	11/11	1'.f51	1".70	44
10	1905	Perrine	P0	8/8	0':18	1'."14	87	8/8	0'."59	0".83	100
11	1906	Pickering	}'4	6/6	4 ','02	3".75	0	6/6	-5".24	9".73	0
12	1906	Pickering	1'4	9/9	-2!'05	8".06	0	9/9	3".39	5'!71	0
13	1906	Barnard	V 0	11/12	-2".58	4".00	21	12/12	0!'12	1".53	67
14	1907	Christie	1'0	15/16	0'."24	0'!72	133	15/16	0".80	0".65	100
15	1908	Perrine	P0	2/2	1".88	0".74	50	2/2	-0".82	0'."03	118
16	1908	Christie	1'0	20/23	0".37	0'."50	167	20/23	-0'."23	0".79	125
17	1909	Christie	P0	11/12	0:'12	1'!13	87	11/12	-0'."32	0'."73	125
18	1910	Christie	1'0	7/7	0':14	1".61	67	7/7	0'."77	0".60	100
19	1912	Barnard	V0	12/12	]'."29	1".22	57	7/7	-0!'18	1".22	80
20	1913	Barnard	V0	5/5	1'."00	0'."77	67	3/3	-0".62	0','32	133
21	1922	VanBiesbroeck	P0	4/5	3".25	0".89	29	4/5	-1".51	1 '."4 o	50
22	1940	Nicholson	1'0	1/1	-0".32	0'."00	200	1/1	-0".04	0".00	200
23	1942	VanBiesbroeck	P0	7/8	1'."73	0".40	56	7/8	-0"22	0'."26	200
24	1952	Bobone	P0	7/7	0'!01	1".03	100	7/7	0".01	0".33	200
25	1955	VanBiesbroeck	1'0	8/11	1".11	0".73	77	8/11	0!'45	0'."48	154
26	1955	VanBiesbroeck	1'0	3/3	2'!35	0".56	44	3/3	0".79	0".89	91
27	1957	VanBiesbroeck	P0	8/8	0".30	0".70	133	8/8	1".27	0'."70	67
28	1960	Roemer	P0	2/2	0".80	0".16	125	2/2	0".39	0".17	25(
29	1968	Chernykh	P0	2/2	0".72	]".9]	5()	2/2	-1".16	0!'51	80
30	1969	VanBiesbroeck	1'0	1/3	1".51	0'."00	67	1/3	0".38	0".00	200
31	1969	VanBiesbroeck	I'o	1/1	-0'."] J	0'."00	200	1/1	0".86	0'."00	100
32	1975	Mulholland	P2	4/5	-0".17	0'."27	333	4/5	() ','07	0".07	500
			1'0	2/3	-0".25	0!'81	133	2/3	-0'."1 6	0'."19	200
33	1981	Bowell	P2	8/10	1".08	0".66	80	8/10	-0".56	()'."85	100
			1'0	2/2	-0'!42	0".57	133	2/2	-1 '."03	0".20	80
34	1981	Debehogne	1'2	20/21	0!'44	0'."43	167	20/21	0!'01	0".58	167
35	1982	Bowell	1'2	2/2	-0'."27	0".63	200	2/2	-0'."95	1".14	80
36	1982	Debehogne	1'2	18/21	-0".31	0'."48	167	18/21	-0".09	0','60	167
37	1992	Rohde	1'0	22/22	1".79	0".36	o	22/22	-0'."8	0".23	0
38	1992	Whipple	1'0	12/12	-0'."19	0'."26	200	12/12	-0".08	0".50	200
3<)	1993	Rohde	}'0	9/9	0'!16	0".26	0	9/9	-1".11	0'!13	0
40	1994	Whipple	1'0	2/2	-0".19	0','72	167	2/2	-0'."4 ()	0".34	200
41	1996	Whipple	C0	21/21	-0':43	0".20	200	21/21	()'."09	0".30	200

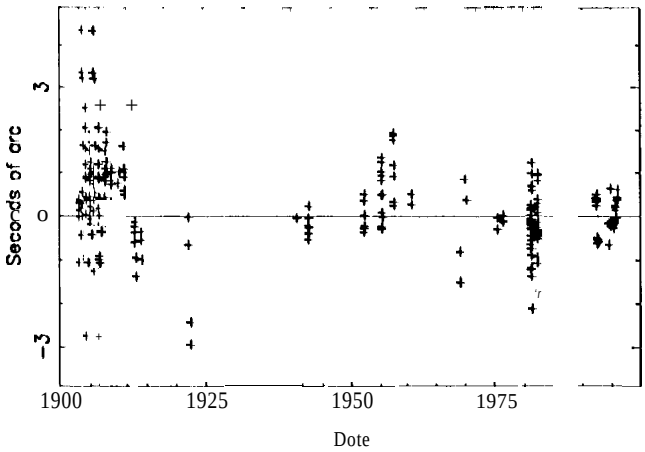

Fig. 2. Declination residuals

vectors are actually provided: the first is a from a fit which used the *complete* dynamical model for post-1966 observations; the second is from a fit which used the *simplified* model. The simplified model was used for the pre-1966 observations in both cases. The RMS differences between the two models over the time period 1899 to **2013 arc** 5.6 km in the radial direction, 50.7 km in the in-orbit direction, and 17.0 km in the out-of-plane direction.

As an aid to those wishing to reproduce the integration, Table 6 contains the state vector for the simplified model at the end of th - integration.

Table 4. Barycentric slate valor at Julian ephemeris date 2439440.50 (1966 Nov. 11 .(I) refered to the Earth mean equator and equinox of J2000

Component	Position (km)	Velocity (kIII/see)
	complete model	
x	-120494$39.8133243500	-0.5851496570455792
y	-2354417.5328888400	1.5137943689486230
z	298479.4052424087	().7872163107138419
	simplified m odel	
x	-12049505.2615439000	-0.5851402385.559751
y	-2354347.6372856380	1.5137971052344600
z	298492.6573990921	0.7872173856992052

5.3. Mean elements

Because an integrated orbit in terms of cartesian coordinates is difficult to interpret geometrically, an alternative qualitative representation of it in the form of mean orbital elements is often useful, Table 7 provides mean elements for the orbit derived by fitting a precessing ellipse model

Table 5. Saturnian system dynamical constants. These are the constants currently being used in the major Saturnian satellite ephemerides, see Jacobson 1996b

Name	Value	Units
Saturn system GM	37940629.764	km^3/sec^2
Mimas GM	2.500	km^3/sec^2
Enceladus GM	4.900	km^3/sec^2
Tethys GM	41.808	km^3/sec^2
Dione GM	73.15(;	km^3/sec^2
Rhea GM	154.000	km^3/sec^2
Titan GM	8978.200	km^3/sec^2
Iapetus GM	106.000	km^3/sec^2
Saturn radius	60330.0	kill
Saturn J_2	162.98X1(1-4	
Saturn $J_2^{\dagger}$	213.74×10^{-4}	
Saturn J_4	-9.1.5X10-4	
Saturn J_6	1.03×10^{-4}	
Saturn pole right ascension	40.58	deg
Saturn pole declination	83.54	deg

† quadra pole for simplified model

Table G. Barycentric slate vector for the simplified model at Julian ephemeris date 2414640.5 (1898 Dec. 17.()) refered to the Earth mean equator and equinox of J2000

Component	Position (km)	Velocity (km/sec)
x	-10038663.710024760	-1.2515248335735 960
y	-6591637.198179223	1.1141266134053140
z	-2665302.021350605	0.6536236440036113

to the integration over the period 1$)()() to 2013. The reference plane is the Phoebe Laplacian plane, the plane on which the orbit precesses almost uniformly. The orientation angles for the Laplacian plane pole are with respect to the Earth mean equator and equinox of J2000 system, and the longitude of the orbit's ascending node is measured from the ascending node of the Laplacian plane on the Earth mean equator of J2000.

6. Accuracy assessment

The accuracy 10 which Phoebe's orbit can be determined is dominated by errors in the observations. The majority of the observations are absolute positions obtained from reductions involving a variety of star catalogues and are subject to relatively large systematic errors due to errors in those catalogues (e.g. zone biases, proper motion). Fundamentally, the errors make it difficult to tie the reference frame of the observation to the IERS/J2000 reference frame of the orbit. Characterizing the uncertainty in the frame-tie is extremely difficult, especially for the older

Table 7. Planetocentric mean orbital elements referred to Phoebe's Laplacian plane

Element	Value	Units
semi-major axis	12944377.64	km
eccentricity	0.16436	
inclination	174.74888	deg
argument of periapsis	330.92048	deg
longitude of ascending node	233.03295	deg
orbital period	548.2127036	days
periapsis rate	0.0020123	deg/day
nodal rate	0.0012489	deg/day
Laplacian plane pole right ascension	275.62887	deg
Laplacian plane pole declination	68.03270	deg

observations. Based on an examination of the residuals we estimate the uncertainty to be no less than 0″.5 for most observation sets although the recent sets from McDonald may be slightly better.

The dominant error source in the observational modelling is the ephemeris of Saturn as it affects the computed absolute position of Phoebe. The uncertainty in the DE 403 Saturn position is about 0″.2.

Relative to the observation related errors, those in the dynamical modelling are quite small. The most important dynamical parameters, the GM's of Saturn and Titan, are well known from the Voyager encounters. Inaccuracies in the ephemerides of the perturbing bodies lead to integration errors of at most a few tens of kilometers.

The effects of the systematic observation errors are alleviated somewhat by the orbit model. The only free parameters in the orbit determination process are the components of the epoch state vector of Phoebe. Consequently, there exist implicit dynamical constraints on the size, shape, and orientation of possible orbits. It is unlikely, assuming the observations are weighted properly, that the orbit has been distorted in an attempt to accommodate a unique systematic error in a particular observation set.

To arrive at a probable accuracy for the orbit, we examined the formal covariance from the data fit, made comparisons with fits to various subsets of the data, made comparisons with fits using differing weighting strategies, and examined sensitivities to various model parameter values. Based on this analysis, the estimate of the 1-σ orbit uncertainties at the time of the planned Cassini flyby (June 12, 2004) appear below:

In-orbit	Radial	Out-of-plane	Period
2500 km	750 km	1000 km	36 sec

The period error translates into a growth in the uncertainty of in-orbit direction of roughly 40 km/yr.

7. Comparison with previous investigations

Rose found a standard error of 1″.52 for the fit to his observation set,; our orbit fits that same set at the 1″.19 level. Except for the Arequipa data, Bykova and Shikhalev and Bec-Borsenberger and Rocher used essentially the same observations. The former give a standard error of 1″.5 (with some edited subset of the Arequipa data included), and the latter quote $\Delta\alpha \cos\delta$ and $\Delta\delta$ errors of 1″.7 and 1″.2, respectively. For the Bec-Borsenberger and Rocher observation set we obtain 1″.66 and 1″.03 for the $\Delta\alpha \cos\delta$ and $\Delta\delta$ errors. It appears, therefore, that we have an orbit which fits the observations of the previous investigators slightly better than their orbits.

We attempted to repeat the other integrations based on the information provided in the publications but were successful only with Rose's orbit (the measure of success was the ability to closely reproduce the published residual statistics). Lack of success in the other two cases is most likely due to the inability of our software to reproduce exactly the perturbations included by the other investigators. One problem with the Bec-Borsenberger and Rocher integration is that they did not give the value of Titan's mass used in their Titan perturbation. We were able to match Rose's orbit because he provided state vectors at both the beginning and end of his integration; we adjusted his initial velocities slightly to ensure that our integration matched his initial and final positions. The RMS differences between our orbit and that of Rose over the time period 1966–2013 are 4769 km in the radial direction, 11 142 km in the in-orbit direction, and 3659 km in the out-of-plane direction.

Bec-Borsenberger and Rocher give sets of polynomial coefficients which can be used to compute geocentric astrometric positions for the years 1981–1990 based on their orbit. From those polynomials we constructed observations for the first of January of each of those years and compared them to computed observations from our orbit. The RMS difference over the ten year period was 0″.48 which suggests agreement with them to better than 3000 km.

8. Conclusions

This article has reported on the determination of the orbit of Phoebe using a numerical integration fit to Earth-based astrometric observations and imaging observations obtained from the Voyager 2 spacecraft. It has included a description of the dynamical models used in the integration, a general discussion of the observations available, an overview of the observation processing procedure, an assessment of the accuracy of the final orbit, and an comparison of the orbit with previously published integrations.

Because of its distance from Saturn, astrometric measures of Phoebe are mostly and will probably continue to be absolute positions. Any future determination of an improved orbit incorporating such positions will require both

high quality observations and careful reduction with new star catalogues closely tied to the IERS/J 2000 reference frame. Re-reduction of the existing observations against such catalogues would also provide significant benefit.

The determination of the orbits of most other planetary satellites relies heavily on intersatellite and planet relative positions, observations which are independent of star catalogue related errors. Among the existing Phoebe observations, there are a number of relative positions, but most are no better than the best absolute positions. New precise wide field simultaneous observations of Phoebe together with Saturn or other Saturnian satellites could provide invaluable relative positions leading to a greatly improved orbit.

An ephemeris based on the orbit described in this article is available electronically from the JPL On-Line Solar System Data Service (Giorgini et al. 1996).

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Appendix: Approximate Titan orbit

The orbit of Titan used in computing the perturbations in the simplified model is defined by a set of mean orbital elements which were derived from a fit to the SAT077 integrated Titan orbit. Because of Titan's small eccentricity and inclination, we adopted the equinoctial form of the elements (Broucke and Cefola 1972); 'Table 8 contains the elements. The table also gives the orientation angles for Titan's Laplacian plane in the Earth mean equator and equinox of J2000 system. The orbital longitudes (λ, ϖ, Ω) are measured from the ascending node of the Laplacian plane on the Earth mean equator of J2000.

When computing the perturbations on Phoebe, the Titan position relative to the Saturnian system barycenter is found with the elements, and then the position of Saturn relative to the barycenter is formed from the Titan position assuming that Titan and Saturn are the only massive bodies in the Saturnian system.

Table 8. Titan barycentric mean orbital elements at Julian ephemeris date 2451179.5 referred Titan's Laplacian plane

Element	Value	Units
a	1221576.576729097	km
$h = e \sin \varpi$	$-1.148469663848269 \times 10^{-2}$	
$k = e \cos \varpi$	$-2.636131692703927 \times 10^{-2}$	
λ	395.6751784158497	deg
$p = \tan(I/2) \sin \Omega$	-2.821 WO061842840X10-3	
$q = \tan(I/2) \cos \Omega$	$-4.100834564469198 \times 10^{-3}$	
$d\varpi/dt$	$1.633780031230999 \times 10^{-8}$	deg/sec
$d\lambda/dt$	2.613075795805477 x10-4	deg/sec
$d\Omega/dt$	$9.384559913818187 \times 10^{-9}$	deg/sec
Laplacian plane pole right ascension	40.58105653650085	deg
Laplacian plane pole declination	83.22516592522730	deg
ϵ	0.02875	
I	0.57043	deg
m	203.54106	deg
Ω	214.53343	deg
Period	15.9454489	days

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