Precision Density Measurements Near the Helium Lambda Transition Using High-Q Microwave Cavities

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Outline:

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- Technical Approach
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- 0 summary

Acknowledgement:

- NASA Contract
- Packard Foundat On

Abstract Submitted
for the MAR96 Meeting of
The American Physical Society

Sorting Category: 9.c

Precision **Density Measurements Near the Helium Lambda Transition Using** High-Q **Microwave Cavities** D. M. STRAYER, Jet Propulsion Laboratory, Caltech, W. JIANG, N.-C. YEH, N. ASPLUND, Caltech — A new experimental approach for high-precision density measurements of liquid helium near the lambda transition is proposed, Using a high-Q Nb microwave cavity ($Q \sim 10^{10}$) and the high-resolution thermometry (HRT), the changes in the density of helium that fills the cavity can be detected to high precision by accurate measurements of the resonant frequency shift (Δf) as a function of the temperature. Since the frequency shift provides direct information for the changes in the dielectric constant, and since the dielectric constant is related to the density through the Clausius-Mossotti relation, the capability of high resolution frequency measurements (to one part in 10^{13}) will enable us to resolve density changes to one part in 10^{10}. Numerical calculations have been performed to demonstrate the feasibility of this approach for mapping out the density profile of liquid helium which couples to the TE modes of a microwave cavity, For temperatures very near the lambda transition, a superfluid-normal fluid interface develops inside the cavity. A numerical deconvolution technique is established to resolve the helium density profile in the cavity. Preliminary experimental data using a TM01 O niobium cavity and with microkelvin temperature resolutions will be presented,
*Supported by NASA contract and Packard Foundation.

[X] Prefer Oral Session
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Date submitted: November 30, 1995 Electronic form version 1.1

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Motivation

- Test of the Critical Relation of β_P near the Lambda Transition

$$\beta_P = A|t|^{-\alpha}(1 + D|t|^x + \cdots) + B$$
$$\equiv -\frac{1}{\rho}\frac{\partial \rho}{\partial T}\bigg|_P$$

High precision measurement of $\rho(T)$

$$\Longrightarrow \beta_P(T) \Longrightarrow \alpha,\ A,\ B, D, x$$

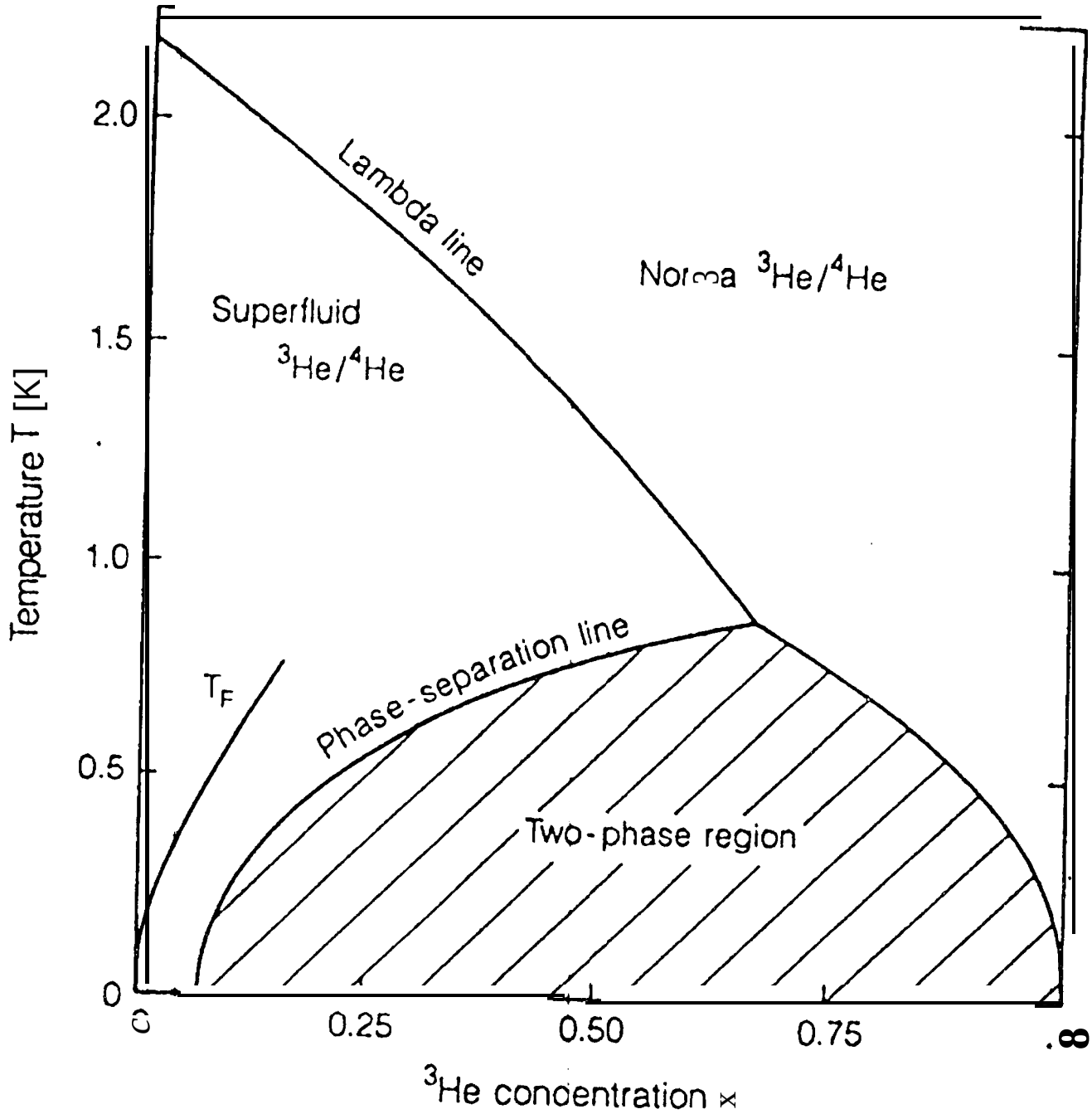
- Related Experimental Work:

	Physical Quantity	Sensitivity ($\epsilon_\lambda \equiv \|1 - (T/T_\lambda)\|$)
Van Degrift & Pellam 1974	Capacitor (C)	$\delta C/C \sim 10^{-9}$, ϵ_λ -10-"5
Muellel, Ahlers, & Pobell 1976	thermal expansion coefficient (β_P)	$\delta P \sim$ 0^{-7} bar, $\epsilon_\lambda \sim 10^{-7}$
Lips, *et al* (to be published inPRL)	heat capacity (C)	$\delta Q/Q \sim 10^{-4}$, $\epsilon_\lambda \sim 10^{-9}$
this technique	resonant frequency shift ($\Delta f/f_0$)	$\delta f/f \sim 10^{-13}$, $\epsilon_\lambda \sim 10^{-9}$

- **Other Applications:**

^3He — ^4He mixture: $\rho_{mix} = x\rho_{^3He} + (1-x)\rho_{^4He}$

Precision measurements of $\rho \Longrightarrow$ precision measurement of x.



Ref. [illegible] Matter and Methods at Low Temperatures
Springer Verlag [illegible]

Technical Approach

- **Clausius-Mossotti Relation** ($\varepsilon \Longleftrightarrow \rho$)

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{4\pi\alpha_0}{3M}\rho$$

ε : dielectric constant
α_0 : polarizability
(assumed to be constant near the lambda transition]])
M : molecular weight
ρ : density

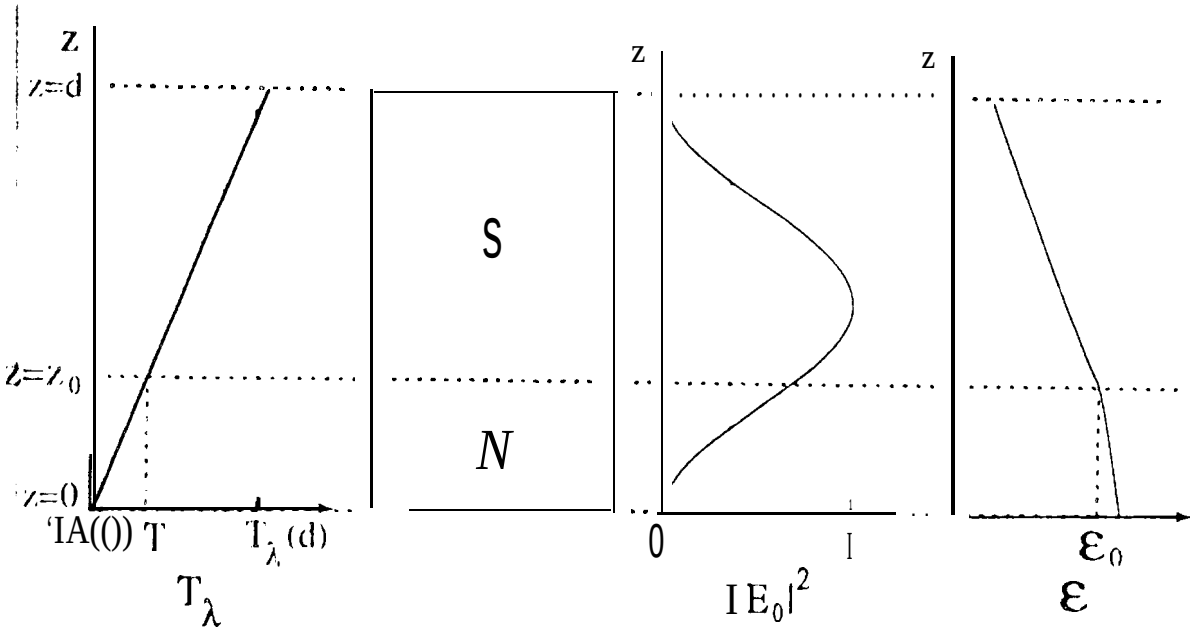
- **In** a microwave cavity, the electric **fields** couple **to** the **dielectric constants** ε

- Cavity **resonant** frequency **shift**(Δf) due to the **small change** of ε of liquid helium:

$$\frac{\Delta f}{f_0} = \frac{f - f_0}{f_0} = \frac{\int_{V_0}(\varepsilon - \varepsilon_0)|E_0|^2 dV}{\int_{V_0}\varepsilon_0|E_0|^2 dV}$$

f_0 : resonant frequency of the resonator at a reference temperature
V_0 : volume of the cavity
E_0 : the electric field of the resonant mode
ε_0 : dielectric constant(s) at the reference temperature

• Gravity [Effect



Consider a cylindrical cavity maintained at a uniform temperature T,

$$T_\lambda(z) = T''(0) - t - \gamma_0 z$$

$$\gamma_0 = 1.273 \times 10^{-6} \mathrm{K/cm}$$

Ref: Ahlers, Phys. Rev. 171, *275* (1968).

If $T_\lambda(0) < T < T_\lambda(d)$, a superfluid/normal fluid interface appears inside the cavity at the position

$$z_0 = \frac{T - T_\lambda(0)}{\gamma_0}$$

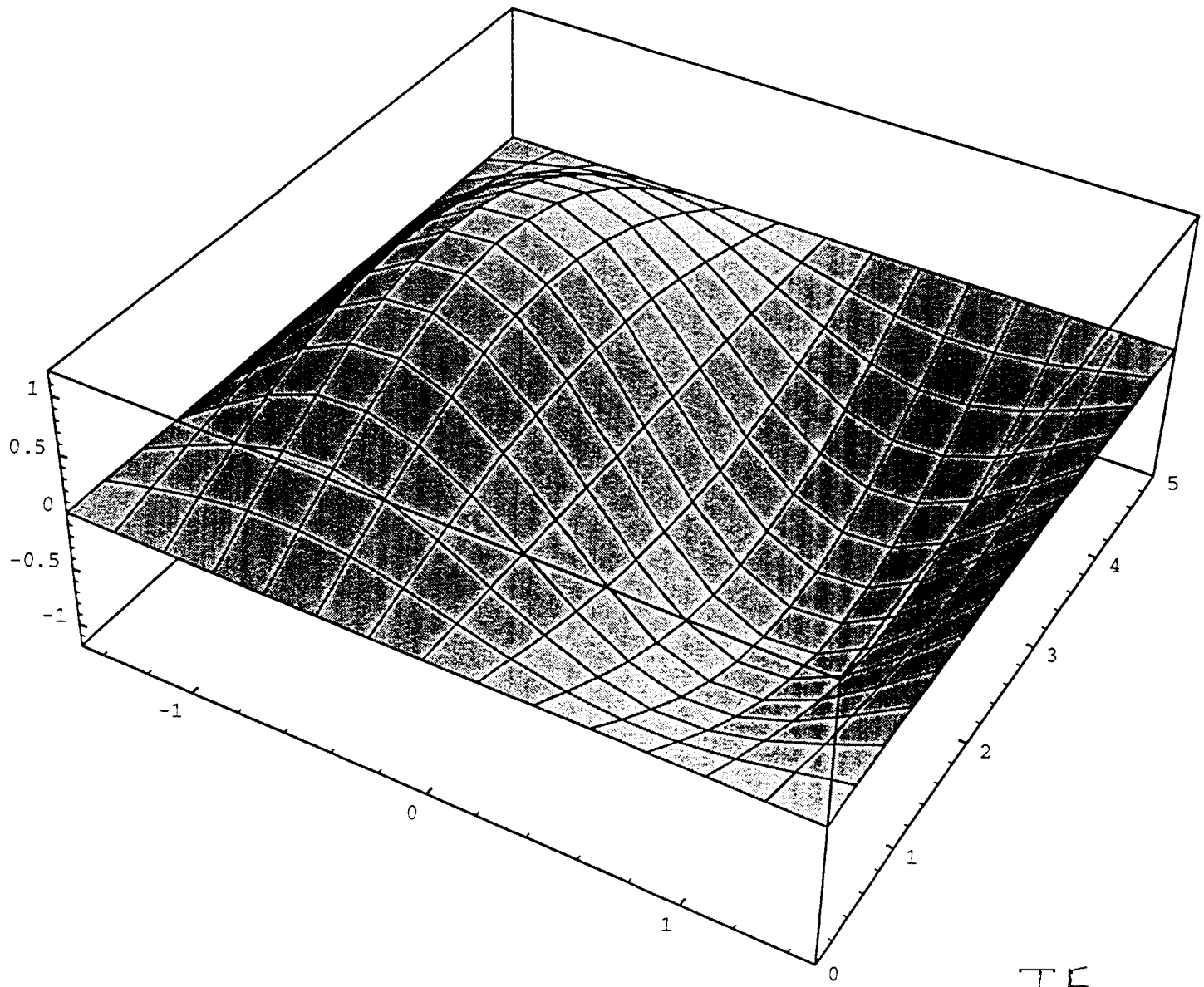
• Deconvolution for Resolving $\rho(z, T)$ near the Lambda Transition

$$\frac{\Delta f}{f_0}(T) \xrightarrow{\text{deconvolution}} \varepsilon(z, T) \xrightarrow[\text{relation}]{\text{Clausius-Mossotti}} \rho(z, T)$$

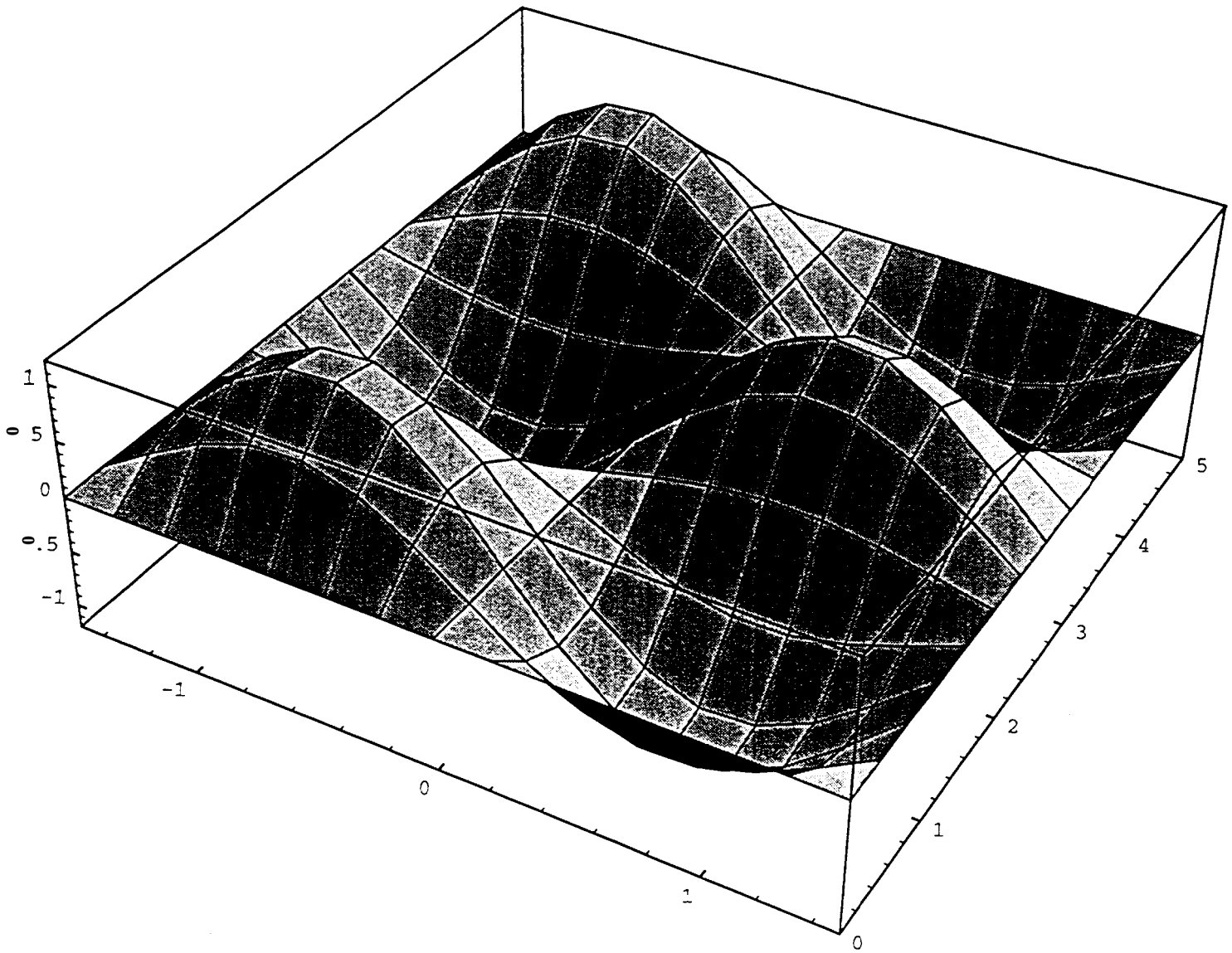
For $TE_{nm\ell}$ mocks,

$$\vec{E} = E_\theta \hat{e}_\theta,\ E_\theta = \frac{ik\eta a H_0}{P'_{nm}} j_n\left(\frac{\bar{P}'_{nm} r}{a}\right) sin\left(\frac{\ell\pi z}{d}\right)$$

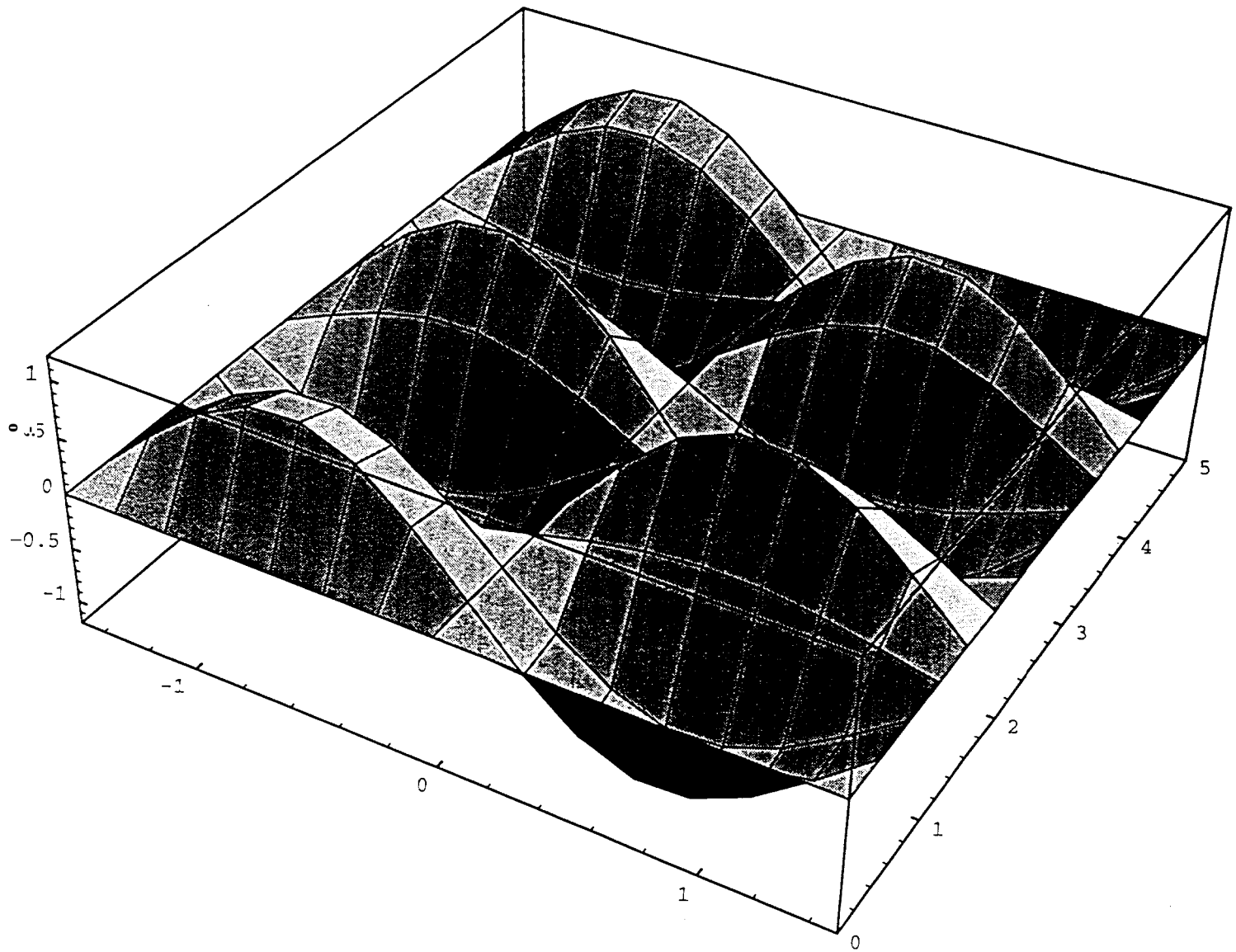
$$\frac{\Delta f}{f_0}(T) = \frac{f - f_0}{f_0} = \frac{\int_0^d [\varepsilon(z,T) - \varepsilon(z, T_\lambda(0)] sin^2\left(\frac{\ell\pi z}{d}\right) dz}{\int_0^d \varepsilon(z, T_\lambda(0)) sin^2\left(\frac{\ell\pi z}{d}\right) dz}$$



TE_0



TE_{013}



TE_{05}

Experimental

- **High-Q Nb** microwave cavity:

  Q $\sim 10^9$ near 2.2 **K**

- **High-Resolution Thermometry (HRT):**

  SQUID magnetic susceptibility:

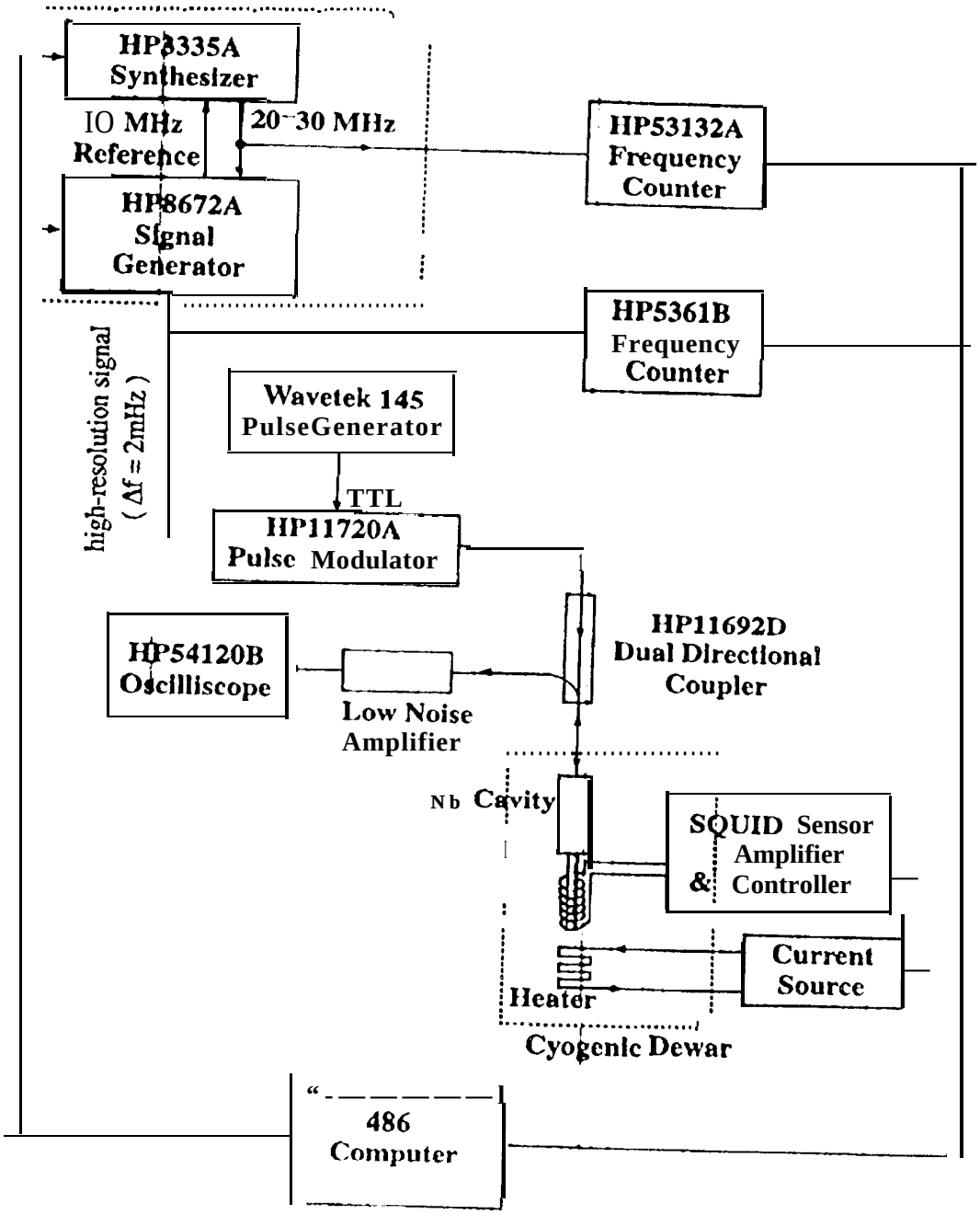
  Ref: Lipa et al, *Physica B+C* 107, *331(1981)*

  Temperature resolution: $\sim 10^{-10}$ **K**
  Temperature stability: $\sim 10^{-9}$ **K**

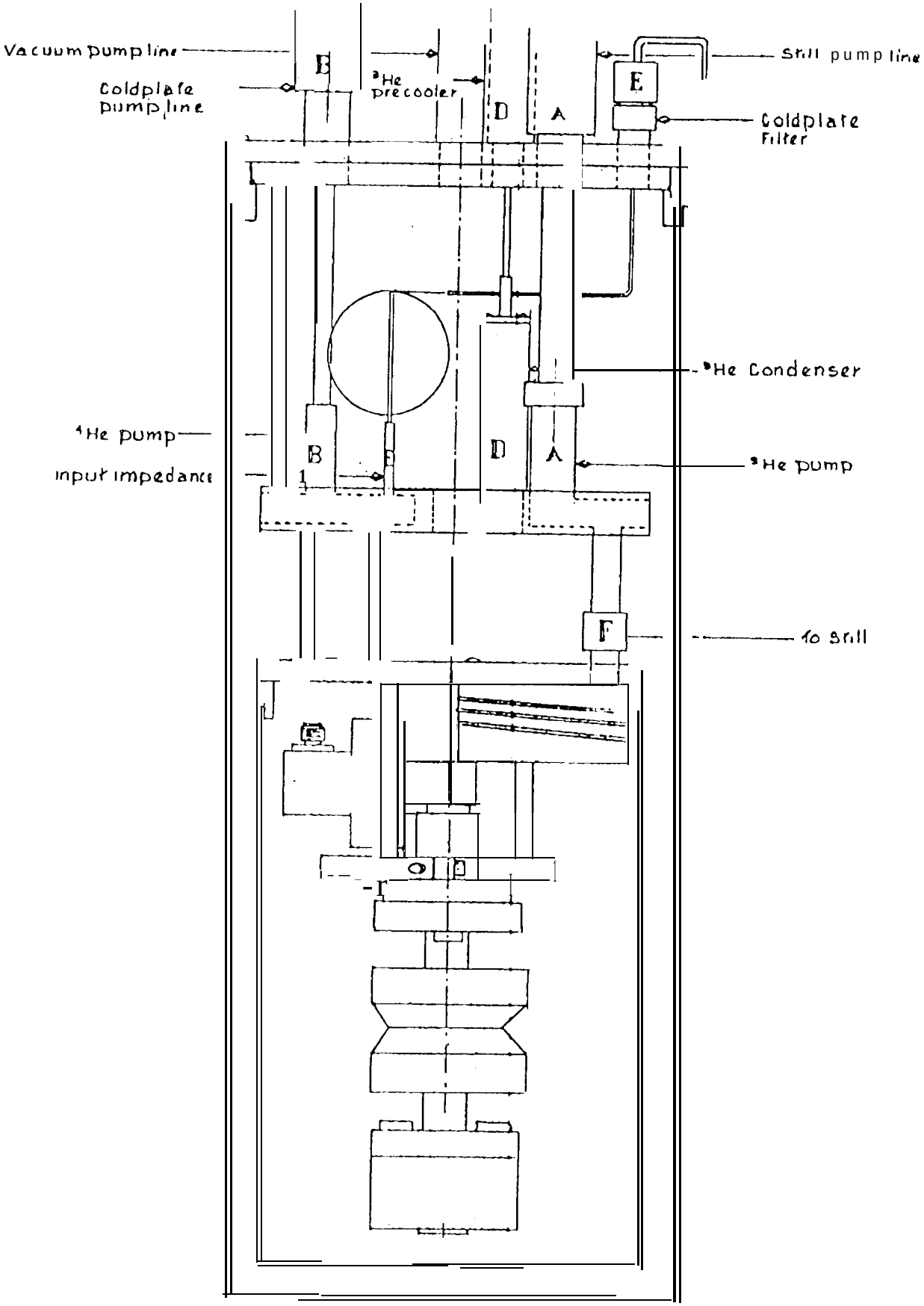
- **High-resolution** frequency source:

  HP high-stability **source:** $\delta f/f \sim 10^{-13}$
  phase-lock loop technique: $\delta f/f \sim 10^{-15}$

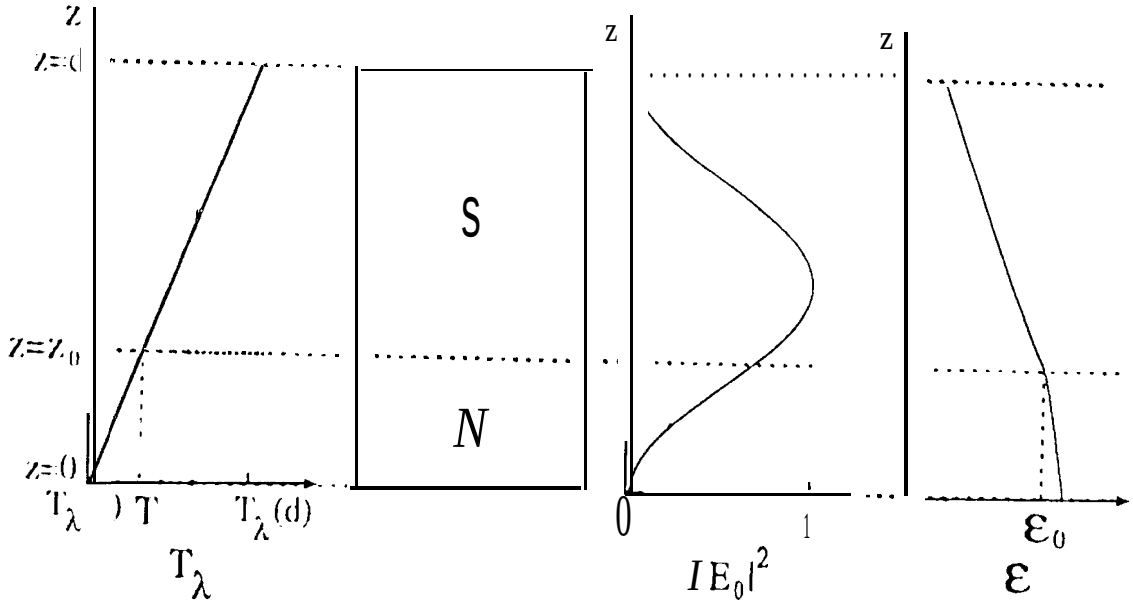
• Block Diagram of Resonant Frequency Measurement Using a High-Q Microwave Cavity



Schematic Measurement Cell Design



Deconvolution Algorithm



For $T_\lambda(0) < T < T_\lambda(z)$,

a given temperature $T \Longrightarrow$ an interface position z_0

Choose

$$T_j = T_\lambda(0) + \gamma_0 z_j, j = 1, \ldots N,$$

such that $z_j = j\frac{d}{N}$

Discretization

$$\frac{\Delta f}{\mathbf{f}_0}(T) = \frac{f - f_0}{f_0} = \frac{\int_0^d [\varepsilon(z,T) - \varepsilon(z, T_\lambda(0))] \sin^2(\frac{\ell \pi z}{d}) dz}{\int_0^d \varepsilon(z, T_\lambda(0)) \sin^2(\frac{\ell \pi z}{d}) dz}$$

$$\Longrightarrow \left[1 - \frac{\Delta f}{f_0}(T_j)\right] \sum_{i=1}^{N} [\varepsilon(t_i^0) - \varepsilon_0] \sin^2\left(\frac{\ell \pi z_i}{d}\right) - \sum_{i=1}^{N} [\varepsilon(t_i^j) - \varepsilon_0] \sin^2\left(\frac{\ell \pi z_i}{d}\right)$$
$$= \frac{\Delta f}{f_0} \frac{\varepsilon_0 N}{2}$$

where

$$\varepsilon_0 \equiv \varepsilon(T = T_\lambda),$$

$$t_i^j = t(T_j, z_i) \equiv T_j - T_\lambda(z_i) = (j - i)\frac{\gamma_0 d}{N}$$

$$z_i = i\frac{d}{N}$$

$$i = 1, ..., N, j = 1, \ldots, N$$

$\Longleftrightarrow$ **Linear** algebra problem

$$\mathrm{A}\ \mathbf{x} = \mathrm{b} \Longrightarrow \mathbf{x} = (A^T A)^{-1} A^T \mathbf{b}$$

where

$$\mathbf{x} \equiv \varepsilon - \varepsilon_0, \quad \mathbf{b} = \frac{\varepsilon_0 N}{2}\mathbf{C}$$

$$\varepsilon = \begin{pmatrix} \varepsilon(-dt) \\ \varepsilon\left(-\frac{N-1}{N}dt\right) \\ \cdot \\ \cdot \\ \varepsilon(dt) \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} \frac{\Delta f}{f_0}(T_1) \\ \frac{\Delta f}{f_0}(T_2) \\ \cdot \\ \cdot \\ \frac{\Delta f}{f_N}(T_N) \end{pmatrix}$$

$$A = \begin{pmatrix} (1-C_1)\mathbf{u}^T & 0 & \dots & 0 \\ (1-C_2)\mathbf{u}^T & 0 & \dots & 0 \\ \cdot & 0 & \dots & 0 \\ \cdot & 0 & \dots & 0 \\ (1-C_N)\mathbf{u}^T & 0 & \dots & 0 \end{pmatrix} - \begin{pmatrix} 0 & \mathbf{u}^T & 0 & \dots & 0 \\ 0 & 0 & \mathbf{u}^T & \dots & 0 \\ \cdot & \dots & \dots & \dots & 0 \\ \cdot & \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & \dots & \mathbf{u}^T \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} \sin^2\left(\frac{\ell\pi z_1}{d}\right) \\ \sin^2\left(\frac{\ell\pi z_2}{d}\right) \\ \cdot \\ \cdot \\ \sin^2\left(\frac{\ell\pi z_N}{d}\right) \end{pmatrix}$$

With z? = 1 & $3, 2N$ unknowns: $\varepsilon - \varepsilon_0$ (assume ε_0 is known) , and $2N$ equations.

• Computer Simulation

$$\rho(t)^{*} = \begin{cases} 0.146081 - 0.1451189 \times 10^{-6.0} \times (2630.9\,t \\ -7768.2\,t\log_{10}|t| + 20547.1\,t^2 + 15963.2\,t\log_{10}|t| \\ +24525.3\,t^3 - 12728\,t^2 t\log_{10}|t|), \text{ for } t < 0; \\ 0.146081, t = 0; \\ 0.146081 + 0.1451189 \times 10^{-6.0} \times (-31288.6\,t \\ -8086.1\,t\log_{10}t^{-}\,]0738.1\,t^2 - 301O.4\,t^3), \text{ for } t > 0. \end{cases}$$

where $t \equiv T - T_\lambda(z)$

$\alpha_0^{*} = 0.123363 N_A = 0.204849\ \mathrm{cm}^3/\mathrm{molecule}$

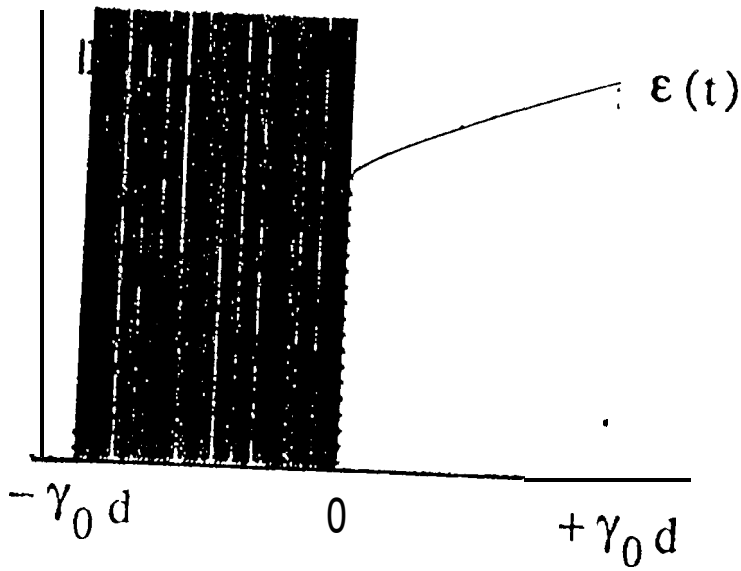
$M^{*} = 6.6424 \times 10^{-24}\mathrm{g}$

$$\Longrightarrow \varepsilon(t) = \frac{1 + 2a\rho}{1 = a\rho}, \quad a \equiv \frac{4\pi\alpha_0}{3M}$$

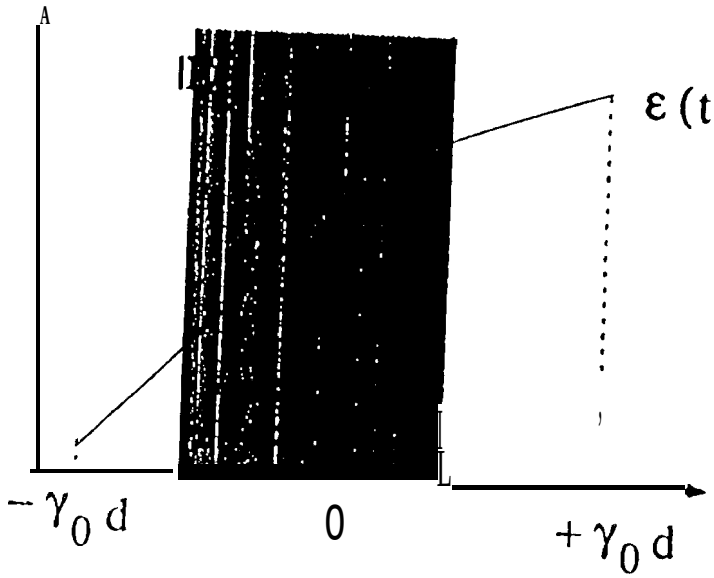
$$\Longrightarrow \frac{\Delta f}{f_0}(T) = \frac{f - f_0}{f_0} = -\frac{\int_0^d [\varepsilon(z,T) - \varepsilon(z,T_\lambda(0))]\sin^2(\frac{\ell\pi z}{d})dz}{\int_0^d \varepsilon(z,T_\lambda(0))\sin^2(\frac{\ell\pi z}{d})dz}$$

* Ref: Donnelly Riegelmann & Barenghi, "The Dielectric Properties of Liquid Helium at the Saturated Vapor Pressure", 1983

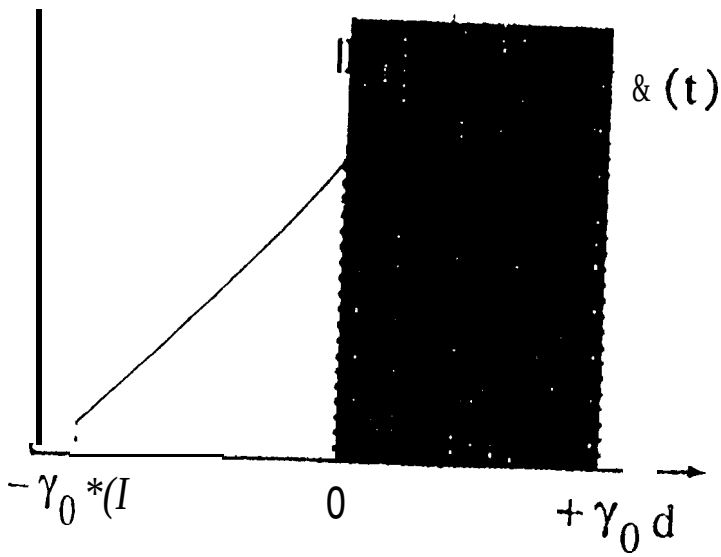
$T = T_\lambda(0)$

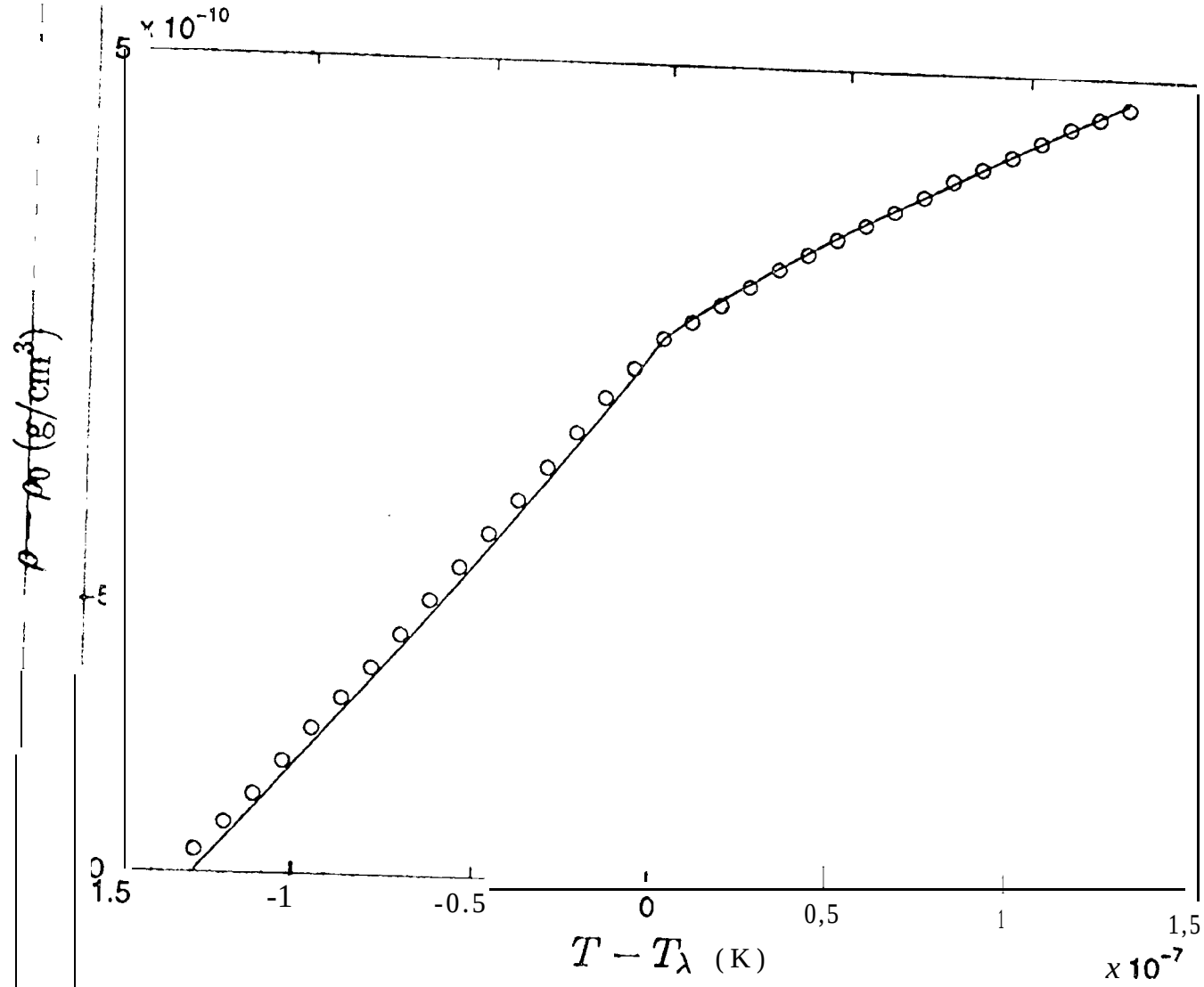


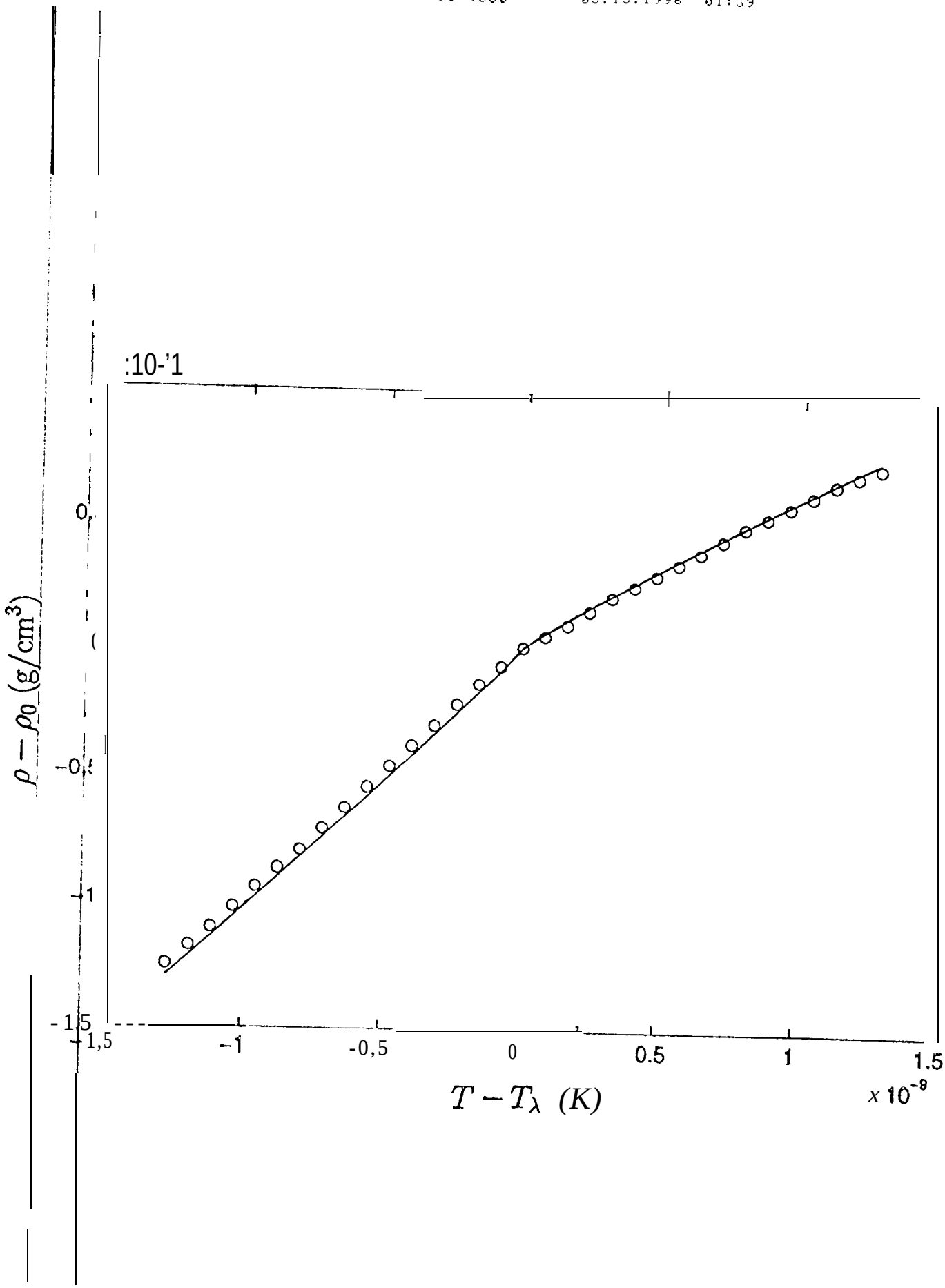
$T_\lambda(0 < T < T_\lambda(d)$

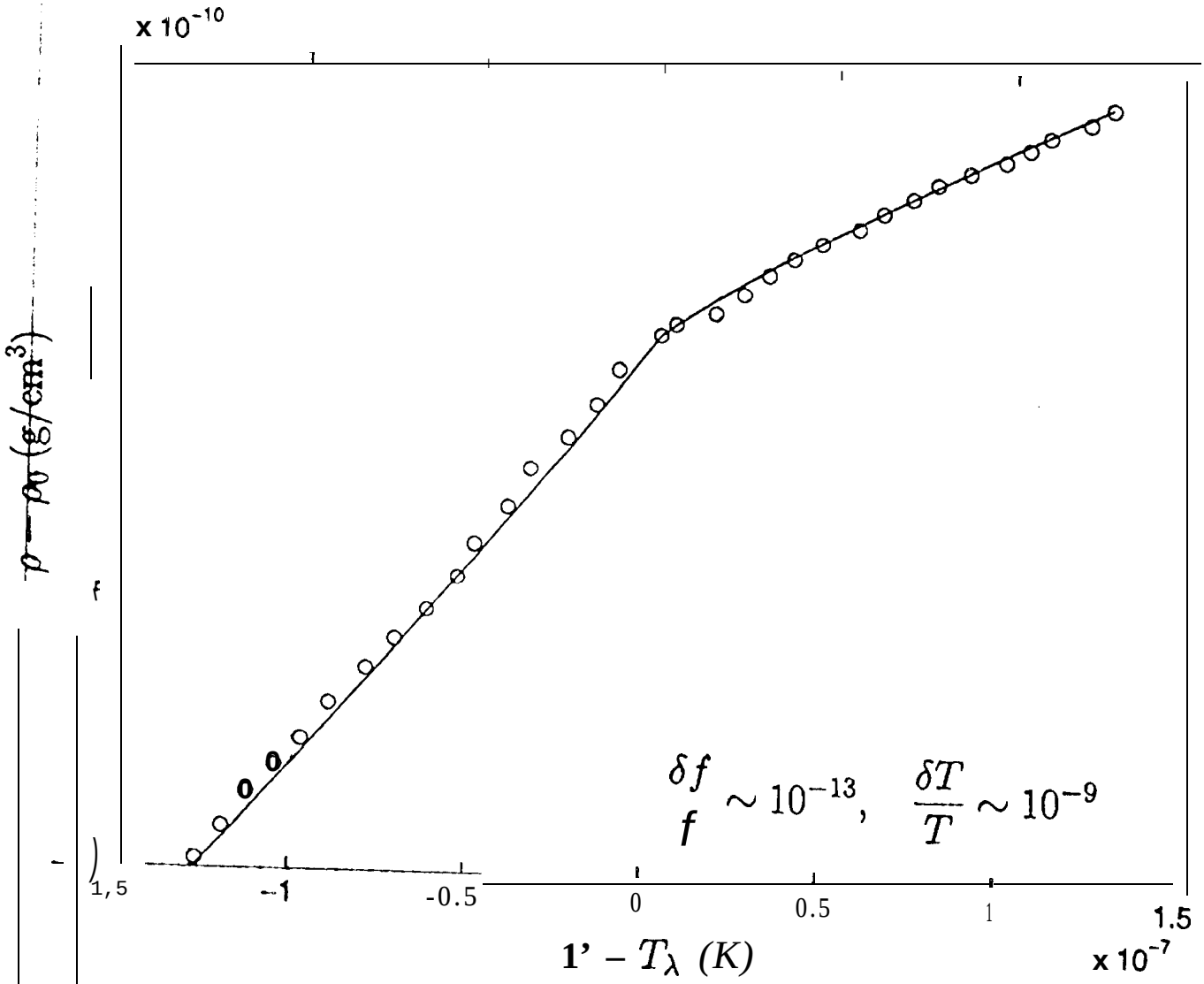


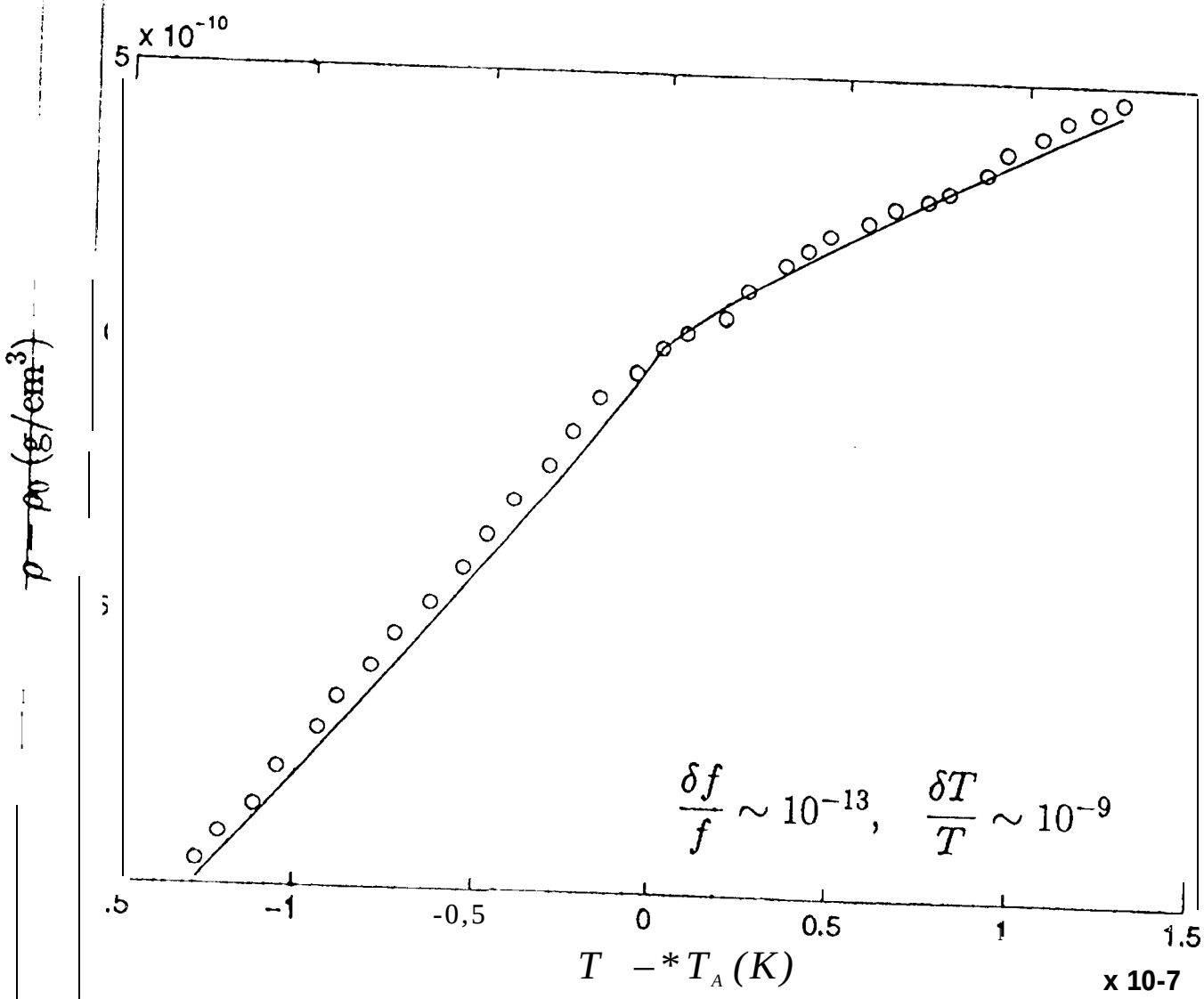
$T \neq T_\lambda(d)$

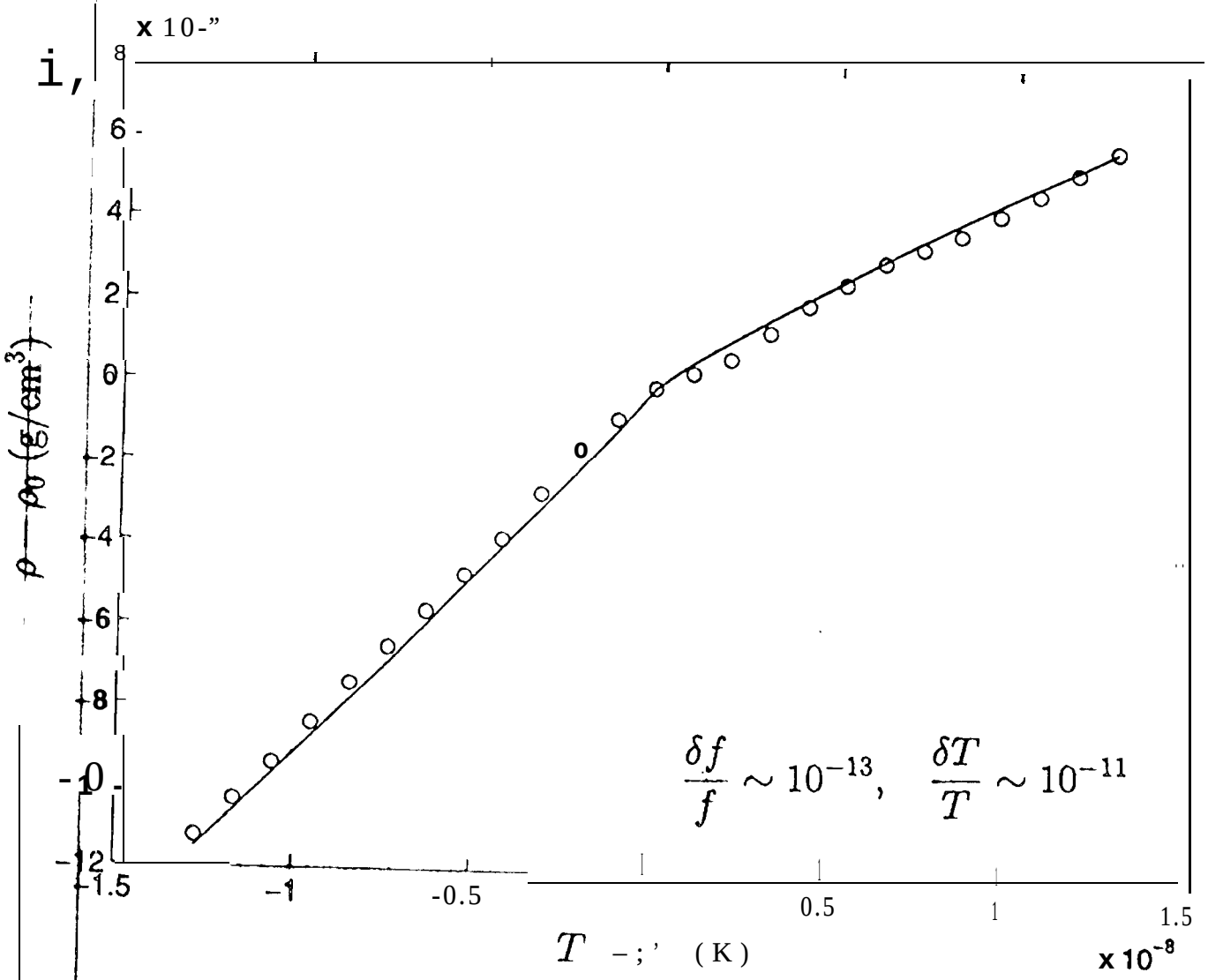


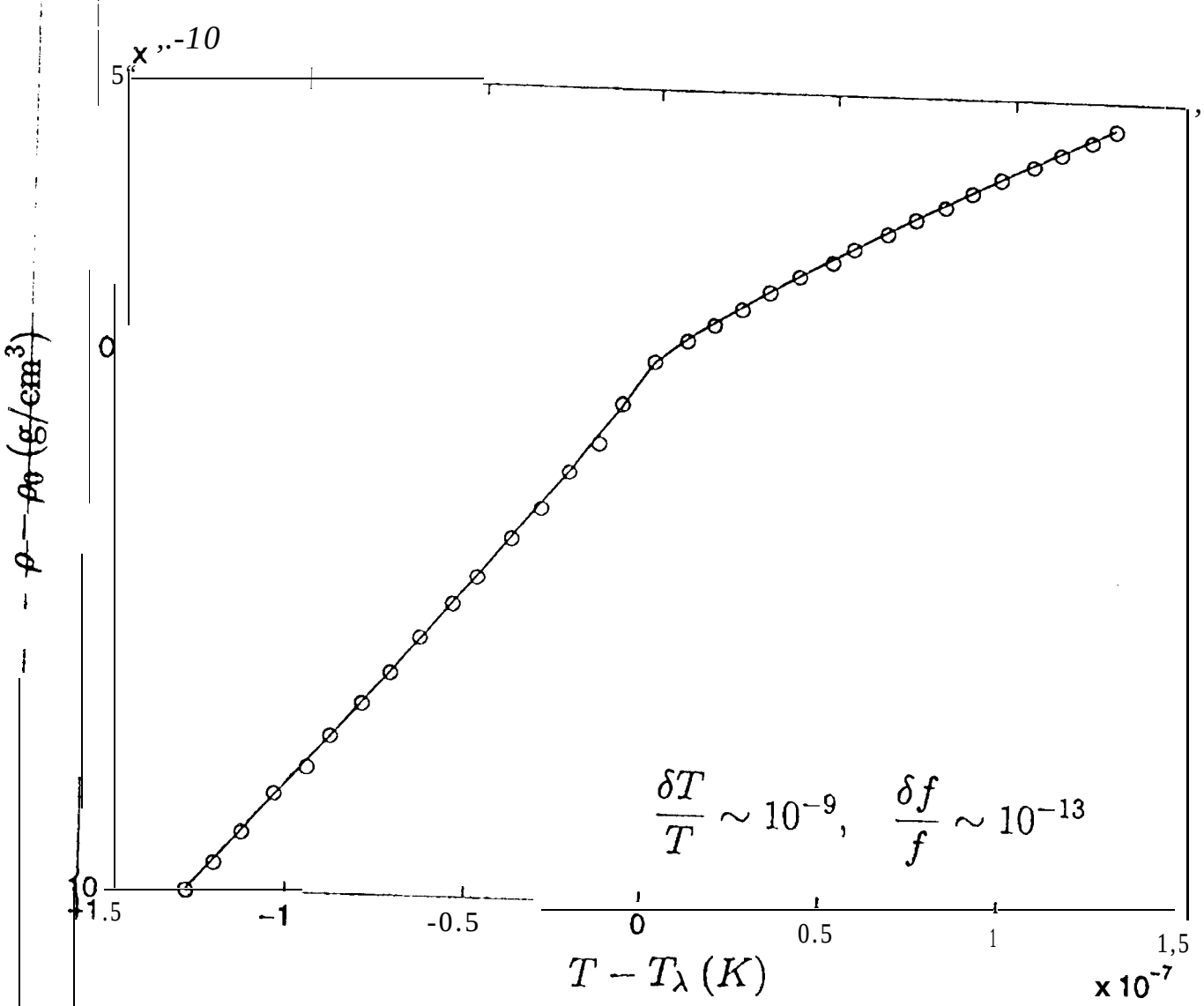

x 10^-10
5
0
1.5
-1
-0.5
0
0,5
1
1,5
x 10^-7
T − T_λ (K)
ρ − ρ_0 (g/cm^3)


:10-'1
0
-0,5
-1
-1,5
-1,5
-1
-0,5
0
0.5
1
1.5
$\rho - \rho_0$ (g/cm^3)
$T - T_\lambda$ (K)
x 10^{-9}

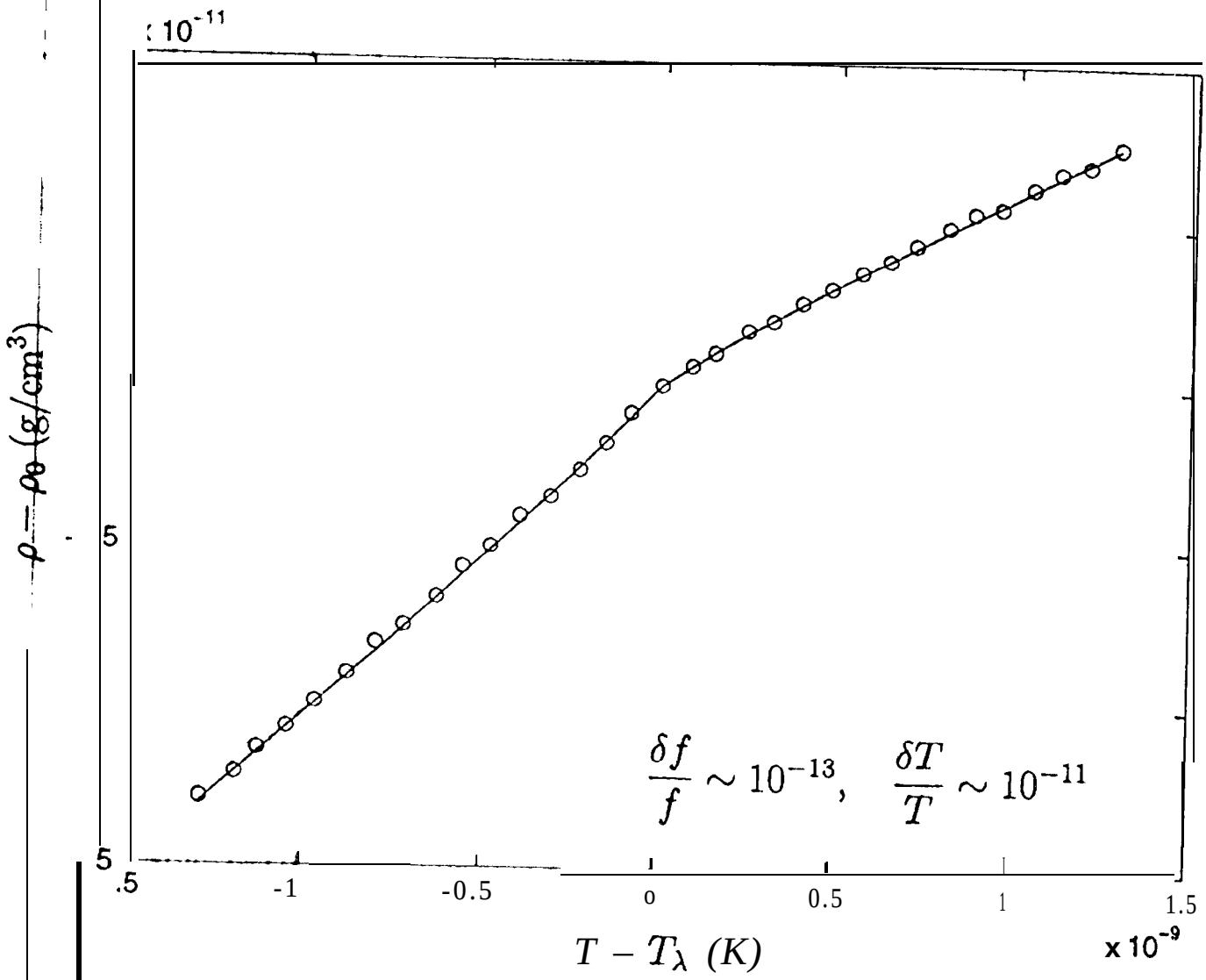

x 10^-10
ρ − ρ0 (g/cm^3)
1,5
-1
-0.5
0
0.5
1
1.5
1' − Tλ (K)
x 10^-7
δf/f ~ 10^-13, δT/T ~ 10^-9


x 10^-10
5
$\rho - \rho_0$ (g/cm^3)
$\frac{\delta f}{f} \sim 10^{-13}, \quad \frac{\delta T}{T} \sim 10^{-9}$
.5
-1
-0,5
0
0.5
1
1.5
$T - T_A$ (K)
x 10-7


x 10-"
8
6
4
2
0
-2
-4
-6
-8
-10
-12
$\rho - \rho_0$ (g/cm^3)
-1.5
-1
-0.5
0.5
1
1.5
T (K)
x 10^{-8}
$\frac{\delta f}{f} \sim 10^{-13}, \quad \frac{\delta T}{T} \sim 10^{-11}$



microgravity environment



Microgravity Environment

- Expected Resolution in Density Measurement

	$\delta f/f$	$\delta T/T$	$6p/p$
earth-bound laboratory	$\sim 10^{-13}$	$\sim 10^{-9}$	$\sim< 3.5 \times 10^{-10}$
	$\sim 10^{-13}$	$\sim 10^{-11}$	$\sim< 0.8 \times 10^{-10}$
microgravity environment	-10^{-13}	$\sim 10^{-9}$	$\sim< 2.5 \times 10^{-10}$
	-10^{-13}	-10^{-11}	$\sim< 2,5 \times 10^{-12}$

Summary

- **High precision** measurements ofhelium density near the **lambda** transition

High-Q Nb cavity ($Q \sim 10^9$ near 2.2 K)
High-resolution thermometry ($|T - T_\lambda| \sim 10^{-9}$ K)
High-resolution frequency source ($\delta f/f \sim 10^{-13}$)

$\Longrightarrow \delta\rho/\rho \sim 10^{-10}$

- **Deconvolution** algorithm

$$\frac{\Delta f}{f_0}(T) \Longrightarrow \epsilon(T, z) \; [\Longrightarrow \rho(T, z)]$$

* **Potential** of the Technique:

Higher precision density measurements in the microgravity environment: $\delta T/T \sim 10^{-11} \Longrightarrow \delta\rho/\rho \sim 10^{-12}$